

What is claimed is:

1. A method of estimating a set of floating ambiguities associated with a set of phase measurements of a plurality n of satellite carrier signals made by a first navigation receiver (B) and a second navigation receiver (R) separated by a distance, wherein a baseline vector (x^o, y^o, z^o) relates the position of the second receiver to the first receiver, each satellite carrier signal being transmitted by a satellite and having a wavelength, wherein each receiver has a time clock for referencing its measurements and wherein any difference between the time clocks may be represented by an offset, said method receiving, for a plurality of two or more time moments j , the following inputs:

a vector γ_j^B representative of a plurality of pseudo-ranges measured by the first navigation receiver (B) and corresponding to the plurality of satellite carrier signals,

a vector γ_j^R representative of a plurality of pseudo-ranges measured by the second navigation receiver (R) and corresponding to the plurality of satellite carrier signals,

a vector D_j^B representative of a plurality of estimated distances between the satellites and the first navigation receiver (B),

a vector D_j^R representative of a plurality of estimated distances between the satellites and the second navigation receiver (R),

a vector ϕ_j^B representative of a plurality of full phase measurements of the satellite carrier signals measured by the first navigation receiver (B),

a vector ϕ_j^R representative of a plurality of full phase measurements of the satellite carrier signals measured by the second navigation receiver (R),

a geometric Jacobian matrix H_j^T whose matrix elements are representative of the changes in the distances between the satellites and one of the receivers that would be caused by changes in that receiver's position and time clock offset, said method comprising the steps of:

(a) generating, for each time moment j , a vector $\Delta\gamma_j$ of a plurality of range residuals of pseudo-range measurements made by the first and second navigation receivers in the form of:

$$\Delta\gamma_j = (\gamma_j^R - \gamma_j^B) - (D_j^R - D_j^B);$$

(b) generating, for each time moment j , a vector $\Delta\varphi_j$ of a plurality of phase residuals of full phase measurements made by the first and second navigation receivers in the form of:

$$\Delta\varphi_j = (\varphi_j^R - \varphi_j^B) - \Lambda^{-1} \cdot (D_j^R - D_j^B),$$

where Λ^{-1} is a diagonal matrix comprising the inverse wavelengths of the satellites;

(c) generating, for time moment $j = 1$, an LU-factorization of a matrix \mathbf{M}_1 or a matrix inverse of matrix \mathbf{M}_1 , the matrix \mathbf{M}_1 being a function of at least Λ^{-1} and \mathbf{H}_1^γ ;

(d) generating, for time moment $j = 1$, a vector \mathbf{N}_1 as a function of at least $\Delta\gamma_1$, $\Delta\varphi_1$, and the LU-factorization of matrix \mathbf{M}_1 or the matrix inverse of matrix \mathbf{M}_1 ;

(e) generating, for an additional time moment $j \neq 1$, an LU-factorization of a matrix \mathbf{M}_j or a matrix inverse of matrix \mathbf{M}_j , the matrix \mathbf{M}_j being a function of at least Λ^{-1} , \mathbf{H}_j^γ and an instance of matrix \mathbf{M} generated for a different time moment; and

(f) generating, for an additional time moment $j \neq 1$, a vector \mathbf{N}_j as a function of at least $\Delta\gamma_j$, $\Delta\varphi_j$, and the LU-factorization or matrix \mathbf{M}_j or the matrix inverse of matrix \mathbf{M}_j , the vector \mathbf{N}_j having estimates of the floating ambiguities.

2. The method of Claim 1 wherein step (c) comprises generating an LU-factorization for a matrix comprising a form equivalent to $(\mathbf{G}^T \mathbf{P}_1 \mathbf{G})$, where:

- the matrix \mathbf{G} has $2n$ rows, n columns, an upper sub-matrix, and a lower sub-matrix, one of the sub-matrices comprises an $n \times n$ zero matrix and the other sub-matrix comprising an $n \times n$ identity matrix,

- the matrix \mathbf{G}^T comprises the transpose matrix of matrix \mathbf{G} , and

- the matrix \mathbf{P}_1 has $2n$ rows, $2n$ columns, and a form which comprises a matrix

$$\text{equivalent to } \left(\mathbf{R}_1^{-1} - \mathbf{R}_1^{-1} \mathbf{Q}_1 (\mathbf{Q}_1^T \mathbf{R}_1^{-1} \mathbf{Q}_1 + q\mathbf{S}_1) \right)^{-1} \mathbf{Q}_1^T \mathbf{R}_1^{-1} \right) \text{ where the matrix } \mathbf{R}_1$$

is a weighting matrix, where the matrix \mathbf{R}_1^{-1} comprises an inverse of matrix \mathbf{R}_1 , where the matrix \mathbf{Q}_1 has an upper sub-matrix and a lower sub-matrix, one of the sub-matrices of \mathbf{Q}_1 comprising matrix \mathbf{H}_1^γ and the other of the sub-matrices of \mathbf{Q}_1

comprising the matrix product $\Lambda^{-1} \mathbf{H}_1^\gamma$, and wherein the matrix \mathbf{Q}_1^T comprises the

transpose of matrix \mathbf{Q}_1 , and where the quantity $q\mathbf{S}_k$ is a zero matrix when the distance

15 between the first and second navigation receivers is unconstrained and where q may be a non-zero weighting parameter and \mathbf{S}_k may be a non-zero matrix when the distance between the first and second navigation receivers is constrained.

3. The method of Claim 2 wherein step (d) comprises the step of generating vector \mathbf{N}_1 to comprise a vector having a form equivalent to $\mathbf{M}_1^{-1}(\mathbf{G}^T \mathbf{P}_1 \boldsymbol{\mu}_1 + q \mathbf{g}_1)$, where the matrix \mathbf{M}_1^{-1} comprises an inverse of matrix of matrix \mathbf{M}_1 , and where the vector $\boldsymbol{\mu}_1$ comprises the vector $[\Delta\gamma_1, \Delta\phi_1]^T$, and where the quantity $q\mathbf{g}_k$ is a zero vector when the distance between the first and second navigation receivers is unconstrained and where q may be non-zero and \mathbf{g}_k may be a non-zero vector when the distance between the first and second navigation receivers is constrained.

4. The method of Claim 1 wherein step (e) comprises generating an LU-factorization for a matrix comprising a form equivalent to $\mathbf{M}_j = \mathbf{M}_{j-1} + \mathbf{G}^T \mathbf{P}_j \mathbf{G}$, where:

- \mathbf{M}_{j-1} comprises the matrix \mathbf{M}_1 of step (c) when $j = 2$ and comprises the matrix \mathbf{M}_j of step (e) for the $j-1$ time moment when $j > 2$,
- the matrix \mathbf{G} has $2n$ rows, n columns, an upper sub-matrix, and a lower sub-matrix, one of the sub-matrices comprising an $n \times n$ zero matrix and the other sub-matrix comprising an $n \times n$ identity matrix,
- the matrix \mathbf{G}^T comprises the transpose matrix of matrix \mathbf{G} , and
- the matrix \mathbf{P}_j has $2n$ rows, $2n$ columns, and a form which comprise a matrix equivalent to $\left(\mathbf{R}_j^{-1} - \mathbf{R}_j^{-1} \mathbf{Q}_j (\mathbf{Q}_j^T \mathbf{R}_j^{-1} \mathbf{Q}_j + q \mathbf{S}_j)^{-1} \mathbf{Q}_j^T \mathbf{R}_j^{-1} \right)$ where the matrix \mathbf{R}_j is a weighting matrix, where the matrix \mathbf{R}_j^{-1} comprises an inverse of matrix \mathbf{R}_j , where the matrix \mathbf{Q}_j has an upper sub-matrix and a lower sub-matrix, one of the sub-matrices of \mathbf{Q}_j comprising matrix \mathbf{H}_j^γ and the other of the sub-matrices of \mathbf{Q}_j comprising the matrix product $\mathbf{A}^{-1} \mathbf{H}_j^\gamma$, and wherein the matrix \mathbf{Q}_j^T comprises the transpose of matrix \mathbf{Q}_j , and where the quantity $q\mathbf{S}_j$ is a zero matrix when the distance between the first and second navigation receivers is unconstrained and where q may

be a non-zero weighting parameter and S_j may be a non-zero matrix when the distance between the first and second navigation receivers is constrained.

20

5. The method of Claim 4 wherein step (f) comprises the step of generating vector N_j to comprise a vector having a form equivalent to

$N_{j-1} + M_j^{-1} [G^T P_j (\mu_j - G N_{j-1}) + q g_j]$, where the matrix M_j^{-1} comprises an inverse of matrix of matrix M_j , where the vector μ_j comprises the vector $[\Delta y_j, \Delta \phi_j]^T$, and

5 where the vector N_{j-1} comprises the vector N_1 generated by step (d) when $j = 2$ and comprises the vector N_{j-1} generated by step (f) for the $j-1$ time moment when $j > 2$, and where the quantity $q g_j$ is a zero vector when the distance between the first and second navigation receivers is unconstrained and where q may be non-zero and g_j may be a non-zero vector when the distance between the first and second navigation receivers is

10 constrained.

6. The method of claim 3 wherein the distance between the first and second navigation receivers is constrained to a distance L_{RB} , wherein step (c) generates matrix S_1 in a form equivalent to:

$$S_1 = \left(1 - \frac{L_{RB}}{\|r_1\|} \right) \begin{pmatrix} I_3 & O_{3 \times 1} \\ O_{1 \times 3} & 0 \end{pmatrix} + \frac{L_{RB}}{\|r_1\|} r_1 r_1^T$$

5 where r_1 is a vector comprising estimates of the three coordinates of the baseline vector for the time moment $j=1$ and a zero as fourth component, where r_1^T is the vector transpose of r_1 , where I_3 is the 3-by-3 identity matrix, where $O_{1 \times 3}$ is a row vector of three zeros, and where $O_{3 \times 1}$ is a column vector of three zeros; and

wherein step (d) generates vector g_1 for the time moment $j=1$ in a form equivalent

10 to:

$$g_1 = G^T R_1^{-1} Q_1 (Q_1^T R_1^{-1} Q_1 + q S_1)^{-1} h_1,$$

where:

$$h_1 = \left(1 - \frac{L_{RB}}{\|r_1\|} \right) r_1.$$

7. The method of Claim 3 wherein weighting matrix \mathbf{R}_l comprises an identity matrix multiplied by a scalar quantity.

8. The method of claim 5 wherein the distance between the first and second navigation receivers is constrained to a distance L_{RB} , wherein step (e) generates matrix \mathbf{S}_j in a form equivalent to:

$$\mathbf{S}_j = \left(1 - \frac{L_{RB}}{\|\mathbf{r}_j\|} \right) \begin{pmatrix} \mathbf{I}_3 & \mathbf{O}_{3 \times 1} \\ \mathbf{O}_{1 \times 3} & 0 \end{pmatrix} + \frac{L_{RB}}{\|\mathbf{r}_j\|} \mathbf{r}_j \mathbf{r}_j^T$$

5 where \mathbf{r}_j is a vector comprising estimates of the three coordinates of the baseline vector for the j-th time moment and a zero as fourth vector component, where \mathbf{r}_j^T is the vector transpose of \mathbf{r}_j , where \mathbf{I}_3 is the 3-by-3 identity matrix, where $\mathbf{O}_{1 \times 3}$ is a row vector of three zeros, and where $\mathbf{O}_{3 \times 1}$ is a column vector of three zeros; and

wherein step (f) generates vector \mathbf{g}_j for the j-th time moment in a form equivalent

10 to:

$$\mathbf{g}_j = \mathbf{G}^T \mathbf{R}_j^{-1} \mathbf{Q}_j (\mathbf{Q}_j^T \mathbf{R}_j^{-1} \mathbf{Q}_j + q \mathbf{S}_j)^{-1} \mathbf{h}_j,$$

where:

$$\mathbf{h}_j = \left(1 - \frac{L_{RB}}{\|\mathbf{r}_j\|} \right) \mathbf{r}_j.$$

9. The method of Claim 5 wherein the weighting matrix \mathbf{R}_j comprises an identity matrix multiplied by a scalar quantity for at least one time moment j.

10. The method of Claim 4 wherein the generation of the LU-factorization in step (e) comprises the steps of:

(g) generating an LU-factorization of matrix \mathbf{M}_{j-1} in a form equivalent to $\mathbf{L}_{j-1} \mathbf{L}_{j-1}^T$ wherein \mathbf{L}_{j-1} is a low-triangular matrix and \mathbf{L}_{j-1}^T is the transpose of \mathbf{L}_{j-1} ;

5 (h) generating a factorization of $\mathbf{G}^T \mathbf{P}_j \mathbf{G}$ in a form equivalent to $\mathbf{T}_j \mathbf{T}_j^T = \mathbf{G}^T \mathbf{P}_j \mathbf{G}$, where \mathbf{T}_j^T is the transpose of \mathbf{T}_j ;

(i) generating an LU-factorization of matrix \mathbf{M}_j in a form equivalent to $\mathbf{L}_j \mathbf{L}_j^T$ from a plurality n of rank-one modifications of matrix \mathbf{L}_{j-1} , each rank-one modification being based on a respective column of matrix \mathbf{T}_j , where n is the number of rows in matrix \mathbf{M}_j .

11. The method of Claim 10 wherein step (h) generates matrix \mathbf{T}_j from a Cholesky factorization of $\mathbf{G}^T \mathbf{P}_j \mathbf{G}$.

12. The method of Claim 10 wherein weighting matrix \mathbf{R}_j has a form equivalent to:

$$\mathbf{R}_j = \begin{bmatrix} \mathbf{R}_j^\gamma & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_j^\varphi \end{bmatrix}, \text{ where } \mathbf{R}^\gamma \text{ and } \mathbf{R}^\varphi \text{ are weighting matrices;}$$

wherein \mathbf{R}^γ and \mathbf{R}^φ are related to a common weighting matrix \mathbf{W} and scaling parameters σ_γ and σ_φ as follows $(\mathbf{R}^\gamma)^{-1} = \frac{1}{\sigma_\gamma^2} \mathbf{W}$, and $(\mathbf{R}^\varphi)^{-1} = \frac{1}{\sigma_\varphi^2} \mathbf{W}$,

wherein step (h) of generating matrix \mathbf{T}_j comprises the steps of:

generating a scalar b in a form equivalent to: $b = \frac{\sigma_\gamma^2 \lambda_{GPS}^2}{\sigma_\gamma^2 + \lambda_{GPS}^2 \sigma_\varphi^2}$, where λ_{GPS} is

the wavelength of the satellite signals,

generating a matrix $\tilde{\mathbf{H}}$ in a form equivalent to $\tilde{\mathbf{H}} = \mathbf{W}^{1/2} \mathbf{H}_j^\gamma$,

generating a Householder matrix \mathbf{S}_{HH} for matrix $\tilde{\mathbf{H}}$, and

generating matrix \mathbf{T}_j in a form equivalent to:

$$\mathbf{T}_j = \frac{1}{\sigma_\varphi} \mathbf{W}^{1/2} \mathbf{S}_{HH} \begin{bmatrix} \sqrt{\left(1 - \frac{b}{\lambda_{GPS}^2}\right)} \mathbf{I}_4 & | & \mathbf{0}_{4 \times (n-4)} \\ \text{---} & | & \text{---} \\ \mathbf{0}_{(n-4) \times 4} & | & \mathbf{I}_{(n-4) \times (n-4)} \end{bmatrix}.$$

13. The method of Claim 10 wherein weighting matrix \mathbf{R}_j is applied to a case where there is a first group of satellite signals having carrier signals in a first wavelength

band and a second group of satellite signals having a carrier signals in a second wavelength band, the weighting frequency having a form equivalent to:

$$5 \quad \mathbf{R}_j = \begin{bmatrix} \mathbf{R}_j^\gamma & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_j^\varphi \end{bmatrix}, \text{ where } \mathbf{R}^\gamma \text{ and } \mathbf{R}^\varphi \text{ are weighting matrices;}$$

wherein \mathbf{R}^γ and \mathbf{R}^φ are related to a common weighting matrix \mathbf{W} , the carrier wavelengths of the first group of signals as represented by matrix $\Lambda^{(1)}$, the carrier wavelengths of the second group of signals as represented by matrix $\Lambda^{(2)}$, the center wavelength of the first band as represented by λ_1 , the center wavelength of the first band as represented by λ_2 , and scaling parameters σ_γ and σ_φ , as follows:

$$\begin{aligned} \left(\mathbf{R}_j^\gamma\right)^{-1} &= \begin{bmatrix} \frac{1}{\sigma_\gamma^2} \mathbf{W} & | & \mathbf{O}_{n \times n} \\ \text{---} & | & \text{---} \\ \mathbf{O}_{n \times n} & | & \frac{1}{\sigma_\gamma^2} \mathbf{W} \end{bmatrix}, \\ \left(\mathbf{R}_j^\varphi\right)^{-1} &= \begin{bmatrix} \frac{1}{\sigma_\varphi^2} \mathbf{W}^{(1)} & | & \mathbf{O}_{n \times n} \\ \text{---} & | & \text{---} \\ \mathbf{O}_{n \times n} & | & \frac{1}{\sigma_\varphi^2} \mathbf{W}^{(2)} \end{bmatrix}, \end{aligned}$$

$$\text{where } \mathbf{W}^{(1)} = \frac{1}{\lambda_1^2} \Lambda^{(1)} \mathbf{W} \Lambda^{(1)}, \quad \mathbf{W}^{(2)} = \frac{1}{\lambda_2^2} \Lambda^{(2)} \mathbf{W} \Lambda^{(2)},$$

15 wherein step (h) of generating matrix \mathbf{T}_j comprises the steps of:

$$\text{generating a scalar } b \text{ in a form equivalent to: } b = \frac{\sigma_\gamma^2 \lambda_1^2 \lambda_2^2}{2\sigma_\varphi^2 \lambda_1^2 \lambda_2^2 + \sigma_\gamma^2 \lambda_1^2 + \sigma_\gamma^2 \lambda_2^2},$$

where λ_1 is the wavelength of a first group of satellite signals and λ_2 is the wavelength of a second group of satellite signals,

$$\text{generating a matrix } \tilde{\mathbf{H}} \text{ in a form equivalent to } \tilde{\mathbf{H}} = \mathbf{W}^{1/2} \mathbf{H}_j^\gamma,$$

20

generating a Householder matrix \mathbf{S}_{HH} for matrix $\tilde{\mathbf{H}}$, and

$$\text{generating matrix } \mathbf{T}_j \text{ in a form equivalent to: } \mathbf{T}_j = \frac{1}{\sigma_\varphi} \begin{bmatrix} \mathbf{A11} & | & \mathbf{O}_{n \times n} \\ \hline \mathbf{A21} & | & \mathbf{A22} \end{bmatrix},$$

where sub-matrixes $\mathbf{A11}$, $\mathbf{A21}$, and $\mathbf{A22}$ are as follows:

$$\begin{aligned} \mathbf{A11} &= \left(\mathbf{W}^{(1)} \right)^{\frac{1}{2}} \mathbf{S}_{\text{HH}} \begin{bmatrix} \sqrt{1-b/\lambda_1^2} \mathbf{I}_4 & | & \mathbf{O}_{4 \times (n-4)} \\ \hline \mathbf{O}_{(n-4) \times 4} & | & \mathbf{I}_{(n-4)} \end{bmatrix}, \\ \mathbf{A21} &= \left(\mathbf{W}^{(2)} \right)^{\frac{1}{2}} \mathbf{S}_{\text{HH}} \begin{bmatrix} -\frac{b}{\lambda_1 \lambda_2 \sqrt{1-b/\lambda_1^2}} \mathbf{I}_4 & | & \mathbf{O}_{4 \times (n-4)} \\ \hline \mathbf{O}_{(n-4) \times 4} & | & \mathbf{O}_{(n-4) \times (n-4)} \end{bmatrix}, \text{ and} \\ \mathbf{A22} &= \left(\mathbf{W}^{(2)} \right)^{\frac{1}{2}} \mathbf{S}_{\text{HH}} \begin{bmatrix} \sqrt{\frac{1-b/\lambda_1^2 - b/\lambda_2^2}{1-b/\lambda_1^2}} \mathbf{I}_4 & | & \mathbf{O}_{4 \times (n-4)} \\ \hline \mathbf{O}_{(n-4) \times 4} & | & \mathbf{I}_{(n-4)} \end{bmatrix}. \end{aligned}$$

25

14. The method of Claim 4 wherein step (d) comprises generating a vector \mathbf{B}_1 to comprise a vector having a form equivalent to $\mathbf{G}^T \mathbf{P}_1 \mu_1 + q\mathbf{g}_1$, where the vector μ_1 comprises the vector $[\Delta\gamma_1, \Delta\varphi_1]^T$, and where the quantity $q\mathbf{g}_j$ is a zero vector when the distance between the first and second navigation receivers is unconstrained and where q may be non-zero and \mathbf{g}_j may be a non-zero vector when the distance between the first and second navigation receivers is constrained; and

5

wherein step (f) further comprises generating, for each time moment $j \neq 1$, a vector \mathbf{B}_j to comprise a matrix having a form equivalent to $\mathbf{B}_{j-1} + \mathbf{G}^T \mathbf{P}_j \mu_j + q\mathbf{g}_j$, where the vector μ_j comprises the vector $[\Delta\gamma_j, \Delta\varphi_j]^T$, and where the vector \mathbf{B}_{j-1} is the vector \mathbf{B}_1 generated by step (d) when $j = 2$ and comprises the vector generated by step (f) for the for the $j-1$ time moment when $j > 2$, and where the quantity $q\mathbf{g}_j$ is a zero vector when the distance between the first and second navigation receivers is unconstrained and where q may be

10

non-zero and \mathbf{g}_j may be a non-zero vector when the distance between the first and second navigation receivers is constrained; and

- 15 wherein step (f) further comprises generating vector \mathbf{N}_j to comprise a vector having a form equivalent to $\mathbf{N}_j = [\mathbf{M}_j]^{-1} \mathbf{B}_j$, where the matrix \mathbf{M}_j^{-1} comprises an inverse of matrix of matrix \mathbf{M}_j .

15. The method of claim 14 wherein the distance between the first and second navigation receivers is constrained to a distance L_{RB} , wherein step (c) generates matrix \mathbf{S}_1 in a form equivalent to:

$$\mathbf{S}_1 = \left(1 - \frac{L_{RB}}{\|\mathbf{r}_1\|} \right) \begin{pmatrix} \mathbf{I}_3 & \mathbf{O}_{3 \times 1} \\ \mathbf{O}_{1 \times 3} & 0 \end{pmatrix} + \frac{L_{RB}}{\|\mathbf{r}_1\|} \mathbf{r}_1 \mathbf{r}_1^T$$

- 5 where \mathbf{r}_1 is a vector comprising estimates of the three coordinates of the baseline vector for the time moment $j=1$ and a zero as fourth component, where \mathbf{r}_1^T is the vector transpose of \mathbf{r}_1 , where \mathbf{I}_3 is the 3-by-3 identity matrix, where $\mathbf{O}_{1 \times 3}$ is a row vector of three zeros, and where $\mathbf{O}_{3 \times 1}$ is a column vector of three zeros; and

- 10 wherein step (d) generates vector \mathbf{g}_1 for the time moment $j=1$ in a form equivalent to:

$$\mathbf{g}_1 = \mathbf{G}^T \mathbf{R}_1^{-1} \mathbf{Q}_1 (\mathbf{Q}_1^T \mathbf{R}_1^{-1} \mathbf{Q}_1 + q \mathbf{S}_1)^{-1} \mathbf{h}_1,$$

where:

$$\mathbf{h}_1 = \left(1 - \frac{L_{RB}}{\|\mathbf{r}_1\|} \right) \mathbf{r}_1.$$

16. The method of claim 14 wherein the distance between the first and second navigation receivers is constrained to a distance L_{RB} , wherein step (e) generates matrix \mathbf{S}_j in a form equivalent to:

$$\mathbf{S}_j = \left(1 - \frac{L_{RB}}{\|\mathbf{r}_j\|} \right) \begin{pmatrix} \mathbf{I}_3 & \mathbf{O}_{3 \times 1} \\ \mathbf{O}_{1 \times 3} & 0 \end{pmatrix} + \frac{L_{RB}}{\|\mathbf{r}_j\|} \mathbf{r}_j \mathbf{r}_j^T$$

- 5 where \mathbf{r}_j is a vector comprising estimates of the three coordinates of the baseline vector for the j -th time moment and a zero as fourth vector component, where \mathbf{r}_j^T is the vector

transpose of \mathbf{r}_j , where \mathbf{I}_3 is the 3-by-3 identity matrix, where $\mathbf{O}_{1 \times 3}$ is a row vector of three zeros, and where $\mathbf{O}_{3 \times 1}$ is a column vector of three zeros; and

10 wherein step (f) generates vector \mathbf{g}_j for the j -th time moment in a form equivalent to:

$$\mathbf{g}_j = \mathbf{G}^T \mathbf{R}_j^{-1} \mathbf{Q}_j (\mathbf{Q}_j^T \mathbf{R}_j^{-1} \mathbf{Q}_j + q \mathbf{S}_j)^{-1} \mathbf{h}_j,$$

where:

$$\mathbf{h}_j = \left(1 - \frac{L_{RB}}{\|\mathbf{r}_j\|} \right) \mathbf{r}_j.$$

17. The method of Claim 14 wherein the weighting matrix \mathbf{R}_j comprises an identity matrix multiplied by a scalar quantity for at least one time moment j .

18. A method of estimating a set of floating ambiguities associated with a set of phase measurements of a plurality n of satellite carrier signals made by a first navigation receiver (B) and a second navigation receiver (R), wherein a baseline vector (x^o, y^o, z^o) relates the position of the second receiver to the first receiver, each satellite carrier signal
5 being transmitted by a satellite and having a wavelength, wherein each receiver has a time clock for referencing its measurements and wherein any difference between the time clocks may be represented by an offset, said method receiving, for a plurality of two or more time moments j , the following inputs for each time moment j :

10 a vector γ_j^B representative of a plurality of pseudo-ranges measured by the first navigation receiver (B) and corresponding to the plurality of satellite carrier signals,

a vector γ_j^R representative of a plurality of pseudo-ranges measured by the second navigation receiver (R) and corresponding to the plurality of satellite carrier signals,

a vector D_j^B representative of a plurality of estimated distances between the satellites and the first navigation receiver (B),

15 a vector D_j^R representative of a plurality of estimated distances between the satellites and the second navigation receiver (R),

a vector ϕ_j^B representative of a plurality of full phase measurements of the satellite carrier signals measured by the first navigation receiver (B),

a vector $\boldsymbol{\varphi}_j^R$ representative of a plurality of full phase measurements of the
 20 satellite carrier signals measured by the second navigation receiver (R),
 a geometric Jacobian matrix \mathbf{H}_j^γ whose matrix elements are representative of the
 changes in the distances between the satellites and one of the receivers that would be
 caused by changes in that receiver's position and time clock offset, said method
 comprising the steps of:

25 (a) generating, for each time moment j , a vector $\Delta\boldsymbol{\gamma}_j$ of a plurality of range
 residuals of pseudo-range measurements made by the first and second navigation
 receivers in the form of: $\Delta\boldsymbol{\gamma}_j = (\boldsymbol{\gamma}_j^R - \boldsymbol{\gamma}_j^B) - (\mathbf{D}_j^R - \mathbf{D}_j^B)$, said step generating a set of
 range residuals $\Delta\boldsymbol{\gamma}_k$, $k=1, \dots, j$;

(b) generating, for each time moment j , a vector $\Delta\boldsymbol{\varphi}_j$ of a plurality of phase
 30 residuals of full phase measurements made by the first and second navigation receivers in
 the form of: $\Delta\boldsymbol{\varphi}_j = (\boldsymbol{\varphi}_j^R - \boldsymbol{\varphi}_j^B) - \boldsymbol{\Lambda}^{-1} \cdot (\mathbf{D}_j^R - \mathbf{D}_j^B)$, where $\boldsymbol{\Lambda}^{-1}$ is a diagonal matrix
 comprising the inverse wavelengths of the satellites, said step generating a set of phase
 residuals $\Delta\boldsymbol{\varphi}_k$, $k=1, \dots, j$;

(c) generating an LU-factorization of a matrix \mathbf{M} or a matrix inverse of matrix \mathbf{M} ,
 35 the matrix \mathbf{M} being a function of at least $\boldsymbol{\Lambda}^{-1}$ and \mathbf{H}_k^γ , for index k of \mathbf{H}_k^γ covering at
 least two of the time moments j ;

(d) generating a vector \mathbf{N} of estimated floating ambiguities as a function of at least
 the set of range residuals $\Delta\boldsymbol{\gamma}_k$, the set of phase residuals $\Delta\boldsymbol{\varphi}_k$, and the LU-factorization of
 matrix \mathbf{M} or the matrix inverse of matrix \mathbf{M} .

19. The method of Claim 18 wherein step (c) comprises generating matrix \mathbf{M} in a
 form equivalent to the summation $\left[\sum_{k=1}^j (\mathbf{G}^T \mathbf{P}_k \mathbf{G}) \right]$,

where:

- the matrix \mathbf{G} has $2n$ rows, n columns, an upper sub-matrix, and a lower sub-matrix,
 5 one of the sub-matrices comprises an $n \times n$ zero matrix and the other sub-matrix
 comprising an $n \times n$ identity matrix,
- the matrix \mathbf{G}^T comprises the transpose matrix of matrix \mathbf{G} , and

- the matrix \mathbf{P}_k has $2n$ rows, $2n$ columns, and a form which comprises a matrix equivalent to $\mathbf{P}_k = \mathbf{R}_k^{-1} - \mathbf{R}_k^{-1} \mathbf{Q}_k (\mathbf{Q}_k^T \mathbf{R}_k^{-1} \mathbf{Q}_k + q\mathbf{S}_k)^{-1} \mathbf{Q}_k^T \mathbf{R}_k^{-1}$, where the matrix \mathbf{R}_k is a weighting matrix, where the matrix \mathbf{R}_k^{-1} comprises an inverse of matrix \mathbf{R}_k , where the matrix \mathbf{Q}_k has an upper sub-matrix and a lower sub-matrix, one of the sub-matrices of \mathbf{Q}_k comprising matrix \mathbf{H}_k^γ and the other of the sub-matrices of \mathbf{Q}_k comprising the matrix product $\mathbf{\Lambda}^{-1} \mathbf{H}_k^\gamma$, and wherein the matrix \mathbf{Q}_k^T comprises the transpose of matrix \mathbf{Q}_k , and where the quantity $q\mathbf{S}_k$ is a zero matrix when the distance between the first and second navigation receivers is unconstrained and where q may be a non-zero weighting parameter and \mathbf{S}_k may be a non-zero matrix when the distance between the first and second navigation receivers is constrained.

20. The method of Claim 19 wherein step (d) comprises generating matrix \mathbf{N} in a form equivalent to:

$$\mathbf{N} = \mathbf{M}^{-1} \times \left[\sum_{k=1}^j (\mathbf{G}^T \mathbf{P}_k \boldsymbol{\mu}_k + q\mathbf{g}_k) \right],$$

- where the matrix \mathbf{M}^{-1} comprises an inverse of matrix of matrix \mathbf{M} , where the vector $\boldsymbol{\mu}_k$ comprises the vector $[\Delta\gamma_k, \Delta\varphi_k]^T$, and where the quantity $q\mathbf{g}_k$ is a zero vector when the distance between the first and second navigation receivers is unconstrained and where q may be non-zero and \mathbf{g}_k may be a non-zero vector when the distance between the first and second navigation receivers is constrained.

21. The method of claim 20 wherein the distance between the first and second navigation receivers is constrained to a distance L_{RB} , wherein step (c) generates matrix \mathbf{S}_k for the k -th time moment in a form equivalent to:

$$\mathbf{S}_k = \left(1 - \frac{L_{RB}}{\|\mathbf{r}_k\|} \right) \begin{pmatrix} \mathbf{I}_3 & \mathbf{O}_{3 \times 1} \\ \mathbf{O}_{1 \times 3} & 0 \end{pmatrix} + \frac{L_{RB}}{\|\mathbf{r}_k\|} \mathbf{r}_k \mathbf{r}_k^T$$

- where \mathbf{r}_k is a vector comprising estimates of the three coordinates of the baseline vector at the k -th time moment, and a zero as fourth component, where \mathbf{r}_k^T is the vector transpose of \mathbf{r}_k , where \mathbf{I}_3 is the 3-by-3 identity matrix, where $\mathbf{O}_{1 \times 3}$ is a row vector of three zeros, and where $\mathbf{O}_{3 \times 1}$ is a column vector of three zeros; and

wherein step (d) generates vector \mathbf{g}_k for the k-th time moment in a form equivalent
 10 to:

$$\mathbf{g}_k = \mathbf{G}^T \mathbf{R}_k^{-1} \mathbf{Q}_k (\mathbf{Q}_k^T \mathbf{R}_k^{-1} \mathbf{Q}_k + q \mathbf{S}_k)^{-1} \mathbf{h}_k,$$

where:

$$\mathbf{h}_k = \left(1 - \frac{L_{RB}}{\|\mathbf{r}_k\|} \right) \mathbf{r}_k.$$

22. The method of Claim 19 wherein at least one of the weighting matrices \mathbf{R}_k comprises an identity matrix multiplied by a scalar quantity.

23. A computer program product for directing a data processor to estimate a set of floating ambiguities associated with a set of phase measurements of a plurality n of satellite carrier signals made by a first navigation receiver (B) and a second navigation receiver (R) separated by a distance, wherein a baseline vector (x^o, y^o, z^o) relates the
 5 position of the second receiver to the first receiver, each satellite carrier signal being transmitted by a satellite and having a wavelength, wherein each receiver has a time clock for referencing its measurements and wherein any difference between the time clocks may be represented by an offset, the process receiving, for a plurality of two or more time moments j , the following inputs:

10 a vector γ_j^B representative of a plurality of pseudo-ranges measured by the first navigation receiver (B) and corresponding to the plurality of satellite carrier signals,

a vector γ_j^R representative of a plurality of pseudo-ranges measured by the second navigation receiver (R) and corresponding to the plurality of satellite carrier signals,

15 a vector \mathbf{D}_j^B representative of a plurality of estimated distances between the satellites and the first navigation receiver (B),

a vector \mathbf{D}_j^R representative of a plurality of estimated distances between the satellites and the second navigation receiver (R),

a vector φ_j^B representative of a plurality of full phase measurements of the satellite carrier signals measured by the first navigation receiver (B),

- 20 a vector φ_j^R representative of a plurality of full phase measurements of the satellite carrier signals measured by the second navigation receiver (R),
- a geometric Jacobian matrix H_j^γ whose matrix elements are representative of the changes in the distances between the satellites and one of the receivers that would be caused by changes in that receiver's position and time clock offset, the computer program
- 25 product comprising:
- a computer-readable medium;
 - a first set of instructions embodied on the computer-readable medium which directs the data processor to generate, for each time moment j , a vector $\Delta\gamma_j$ of a plurality of range residuals of pseudo-range measurements made by the first and second navigation
 - 30 receivers in the form of: $\Delta\gamma_j = (\gamma_j^R - \gamma_j^B) - (D_j^R - D_j^B)$;
 - a second set of instructions embodied on the computer-readable medium which directs the data processor to generate, for each time moment j , a vector $\Delta\varphi_j$ of a plurality of phase residuals of full phase measurements made by the first and second navigation receivers in the form of: $\Delta\varphi_j = (\varphi_j^R - \varphi_j^B) - \Lambda^{-1} \cdot (D_j^R - D_j^B)$, where Λ^{-1} is a diagonal
 - 35 matrix comprising the inverse wavelengths of the satellites;
 - a third set of instructions embodied on the computer-readable medium which directs the data processor to generate, for time moment $j = 1$, an LU-factorization of a matrix M_1 or a matrix inverse of matrix M_1 , the matrix M_1 being a function of at least Λ^{-1} and H_1^γ ;
 - 40 a fourth set of instructions embodied on the computer-readable medium which directs the data processor to generate, for time moment $j = 1$, a vector N_1 as a function of at least $\Delta\gamma_1$, $\Delta\varphi_1$, and the LU-factorization of matrix M_1 or the matrix inverse of matrix M_1 ;
 - a fifth set of instructions embodied on the computer-readable medium which
 - 45 directs the data processor to generate, for an additional time moment $j \neq 1$, an LU-factorization of a matrix M_j or a matrix inverse of matrix M_j , the matrix M_j being a function of at least Λ^{-1} and H_j^γ ; and
 - a sixth set of instructions embodied on the computer-readable medium which directs the data processor to generate, for an additional time moment $j \neq 1$, a vector N_j as a

50 function of at least $\Delta\gamma_j$, $\Delta\phi_j$, and the LU-factorization or matrix \mathbf{M}_j or the matrix inverse of matrix \mathbf{M}_j , the vector \mathbf{N}_j having estimates of the floating ambiguities.

24. The computer program product of Claim 23 wherein the third set of instructions directs the data processor to generate an LU-factorization for a matrix comprising a form equivalent to $(\mathbf{G}^T \mathbf{P}_1 \mathbf{G})$, where:

- the matrix \mathbf{G} has $2n$ rows, n columns, an upper sub-matrix, and a lower sub-matrix,
- 5 one of the sub-matrices comprises an $n \times n$ zero matrix and the other sub-matrix comprising an $n \times n$ identity matrix,
- the matrix \mathbf{G}^T comprises the transpose matrix of matrix \mathbf{G} , and
- the matrix \mathbf{P}_1 has $2n$ rows, $2n$ columns, and a form which comprises a matrix equivalent to $\left(\mathbf{R}_1^{-1} - \mathbf{R}_1^{-1} \mathbf{Q}_1 (\mathbf{Q}_1^T \mathbf{R}_1^{-1} \mathbf{Q}_1 + q \mathbf{S}_1)^{-1} \mathbf{Q}_1^T \mathbf{R}_1^{-1} \right)$ where the matrix \mathbf{R}_1
- 10 is a weighting matrix, where the matrix \mathbf{R}_1^{-1} comprises an inverse of matrix \mathbf{R}_1 , where the matrix \mathbf{Q}_1 has an upper sub-matrix and a lower sub-matrix, one of the sub-matrices of \mathbf{Q}_1 comprising matrix \mathbf{H}_1^γ and the other of the sub-matrices of \mathbf{Q}_1 comprising the matrix product $\mathbf{\Lambda}^{-1} \mathbf{H}_1^\gamma$, and wherein the matrix \mathbf{Q}_1^T comprises the transpose of matrix \mathbf{Q}_1 , and where the quantity $q \mathbf{S}_k$ is a zero matrix when the distance
- 15 between the first and second navigation receivers is unconstrained and where q may be a non-zero weighting parameter and \mathbf{S}_k may be a non-zero matrix when the distance between the first and second navigation receivers is constrained.

25. The computer program product of Claim 24 wherein the fourth set of instructions directs the data processor to generate vector \mathbf{N}_1 to comprise a vector having a form equivalent to $\mathbf{M}_1^{-1} (\mathbf{G}^T \mathbf{P}_1 \boldsymbol{\mu}_1 + q \mathbf{g}_1)$, where the matrix \mathbf{M}_1^{-1} comprises an inverse of matrix of matrix \mathbf{M}_1 , and where the vector $\boldsymbol{\mu}_1$ comprises the vector $[\Delta\gamma_1, \Delta\phi_1]^T$, and

5 where the quantity $q \mathbf{g}_k$ is a zero vector when the distance between the first and second navigation receivers is unconstrained and where q may be non-zero and \mathbf{g}_k may be a non-zero vector when the distance between the first and second navigation receivers is constrained.

26. The computer program product of Claim 23 wherein the fifth set of instructions directs the data processor to generate an LU-factorization for a matrix comprising a form equivalent to $\mathbf{M}_j = \mathbf{M}_{j-1} + \mathbf{G}^T \mathbf{P}_j \mathbf{G}$, where:

- \mathbf{M}_{j-1} comprises the matrix \mathbf{M}_1 of step (c) when $j = 2$ and comprises the matrix \mathbf{M}_j of step (e) for the $j-1$ time moment when $j > 2$,
- the matrix \mathbf{G} has $2n$ rows, n columns, an upper sub-matrix, and a lower sub-matrix, one of the sub-matrices comprising an $n \times n$ zero matrix and the other sub-matrix comprising an $n \times n$ identity matrix,

- the matrix \mathbf{G}^T comprises the transpose matrix of matrix \mathbf{G} , and

- the matrix \mathbf{P}_j has $2n$ rows, $2n$ columns, and a form which comprise a matrix

equivalent to $\left(\mathbf{R}_j^{-1} - \mathbf{R}_j^{-1} \mathbf{Q}_j \left(\mathbf{Q}_j^T \mathbf{R}_j^{-1} \mathbf{Q}_j + q \mathbf{S}_j \right)^{-1} \mathbf{Q}_j^T \mathbf{R}_j^{-1} \right)$ where the matrix

\mathbf{R}_j is a weighting matrix, where the matrix \mathbf{R}_j^{-1} comprises an inverse of matrix \mathbf{R}_j , where the matrix \mathbf{Q}_j has an upper sub-matrix and a lower sub-matrix, one of the sub-matrices of \mathbf{Q}_j comprising matrix \mathbf{H}_j^T and the other of the sub-matrices of \mathbf{Q}_j

- comprising the matrix product $\mathbf{\Lambda}^{-1} \mathbf{H}_j^T$, and wherein the matrix \mathbf{Q}_j^T comprises the transpose of matrix \mathbf{Q}_j , and where the quantity $q \mathbf{S}_j$ is a zero matrix when the distance between the first and second navigation receivers is unconstrained and where q may be a non-zero weighting parameter and \mathbf{S}_j may be a non-zero matrix when the distance between the first and second navigation receivers is constrained.

27. The computer program product of Claim 26 wherein the sixth set of instructions directs the data processor to generate a vector \mathbf{N}_j to comprise a vector having a form equivalent to $\mathbf{N}_{j-1} + \mathbf{M}_j^{-1} \left[\mathbf{G}^T \mathbf{P}_j \left(\boldsymbol{\mu}_j - \mathbf{G} \mathbf{N}_{j-1} \right) + q \mathbf{g}_j \right]$, where the matrix \mathbf{M}_j^{-1} comprises an inverse of matrix of matrix \mathbf{M}_j , where the vector $\boldsymbol{\mu}_j$ comprises the vector $[\Delta \gamma_j, \Delta \phi_j]^T$, and where the vector \mathbf{N}_{j-1} comprises the vector \mathbf{N}_1 generated by step (d) when $j = 2$ and comprises the vector \mathbf{N}_{j-1} generated by step (f) for the $j-1$ time moment when $j > 2$, and where the quantity $q \mathbf{g}_j$ is a zero vector when the distance between the first and second navigation receivers is unconstrained and where q may be non-zero and

10 \mathbf{g}_j may be a non-zero vector when the distance between the first and second navigation receivers is constrained.

28. The computer program product of Claim 27 wherein the distance between the first and second navigation receivers is constrained to a distance L_{RB} , and wherein the fifth set of instructions directs the data processor to generate matrix \mathbf{S}_j in a form equivalent to:

$$5 \quad \mathbf{S}_j = \left(1 - \frac{L_{RB}}{\|\mathbf{r}_j\|} \right) \begin{pmatrix} \mathbf{I}_3 & \mathbf{O}_{3 \times 1} \\ \mathbf{O}_{1 \times 3} & 0 \end{pmatrix} + \frac{L_{RB}}{\|\mathbf{r}_j\|} \mathbf{r}_j \mathbf{r}_j^T$$

where \mathbf{r}_j is a vector comprising estimates of the three coordinates of the baseline vector for the j -th time moment and a zero as fourth vector component, where \mathbf{r}_j^T is the vector transpose of \mathbf{r}_j , where \mathbf{I}_3 is the 3-by-3 identity matrix, where $\mathbf{O}_{1 \times 3}$ is a row vector of three zeros, and where $\mathbf{O}_{3 \times 1}$ is a column vector of three zeros; and

10 wherein the sixth set of instructions directs the data processor to generate vector \mathbf{g}_j for the j -th time moment in a form equivalent to:

$$\mathbf{g}_j = \mathbf{G}^T \mathbf{R}_j^{-1} \mathbf{Q}_j (\mathbf{Q}_j^T \mathbf{R}_j^{-1} \mathbf{Q}_j + q \mathbf{S}_j)^{-1} \mathbf{h}_j,$$

where:

$$\mathbf{h}_j = \left(1 - \frac{L_{RB}}{\|\mathbf{r}_j\|} \right) \mathbf{r}_j.$$

15

29. The computer program product of Claim 28 wherein the fifth set of instructions comprises:

a seventh set of instructions that direct the data processor to generate an LU-factorization of matrix \mathbf{M}_{j-1} in a form equivalent to $\mathbf{L}_{j-1} \mathbf{L}_{j-1}^T$ wherein \mathbf{L}_{j-1} is a low-triangular matrix and \mathbf{L}_{j-1}^T is the transpose of \mathbf{L}_{j-1} ;

5

an eighth set of instructions that direct the data processor to generate a factorization of $\mathbf{G}^T \mathbf{P}_j \mathbf{G}$ in a form equivalent to $\mathbf{T}_j \mathbf{T}_j^T = \mathbf{G}^T \mathbf{P}_j \mathbf{G}$, where \mathbf{T}_j^T is the transpose of \mathbf{T}_j ; and

a ninth set of instructions that direct the data processor to generate an LU-factorization of matrix \mathbf{M}_j in a form equivalent to $\mathbf{L}_j \mathbf{L}_j^T$ from a plurality n of rank-one

10

modifications of matrix \mathbf{L}_{j-1} , each rank-one modification being based on a respective column of matrix \mathbf{T}_j , where n is the number of rows in matrix \mathbf{M}_j .

30. The computer program product of Claim 29 wherein the eighth set of instructions directs the data processor to generate matrix \mathbf{T}_j from a Cholesky factorization of $\mathbf{G}^T \mathbf{P}_j \mathbf{G}$.

31. The computer program product of Claim 29 wherein weighting matrix \mathbf{R}_j has a form equivalent to:

$$\mathbf{R}_j = \begin{bmatrix} \mathbf{R}_j^\gamma & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_j^\phi \end{bmatrix}, \text{ where } \mathbf{R}^\gamma \text{ and } \mathbf{R}^\phi \text{ are weighting matrices;}$$

wherein \mathbf{R}^γ and \mathbf{R}^ϕ are related to a common weighting matrix \mathbf{W} and scaling parameters σ_γ and σ_ϕ as follows $(\mathbf{R}^\gamma)^{-1} = \frac{1}{\sigma_\gamma^2} \mathbf{W}$, and $(\mathbf{R}^\phi)^{-1} = \frac{1}{\sigma_\phi^2} \mathbf{W}$; and

wherein the eighth set of instructions comprises:

instructions that direct the data processor to generate a scalar b in a form

equivalent to: $b = \frac{\sigma_\gamma^2 \lambda_{GPS}^2}{\sigma_\gamma^2 + \lambda_{GPS}^2 \sigma_\phi^2}$, where λ_{GPS}^2 is the wavelength of the satellite signals,

instructions that direct the data processor to generate a matrix $\tilde{\mathbf{H}}$ in a form

equivalent to $\tilde{\mathbf{H}} = \mathbf{W}^{1/2} \mathbf{H}_j^\gamma$,

instructions that direct the data processor to generate a Householder matrix \mathbf{S}_{HH} for matrix $\tilde{\mathbf{H}}$, and

instructions that direct the data processor to generate matrix \mathbf{T}_j in a form equivalent to:

$$\mathbf{T}_j = \frac{1}{\sigma_\phi} \mathbf{W}^{1/2} \mathbf{S}_{HH} \begin{bmatrix} \sqrt{\left(1 - \frac{b}{\lambda_{GPS}^2}\right)} \mathbf{I}_4 & | & \mathbf{0}_{4 \times (n-4)} \\ \text{---} & | & \text{---} \\ \mathbf{0}_{(n-4) \times 4} & | & \mathbf{I}_{(n-4) \times (n-4)} \end{bmatrix}.$$

32. The method of Claim 29 wherein weighting matrix \mathbf{R}_j is applied to a case where there is a first group of satellite signals having carrier signals in a first wavelength band and a second group of satellite signals having a carrier signals in a second wavelength band, the weighting frequency having a form equivalent to:

$$5 \quad \mathbf{R}_j = \begin{bmatrix} \mathbf{R}_j^\gamma & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_j^\varphi \end{bmatrix}, \text{ where } \mathbf{R}^\gamma \text{ and } \mathbf{R}^\varphi \text{ are weighting matrices;}$$

wherein \mathbf{R}^γ and \mathbf{R}^φ are related to a common weighting matrix \mathbf{W} , the carrier wavelengths of the first group of signals as represented by matrix $\Lambda^{(1)}$, the carrier wavelengths of the second group of signals as represented by matrix $\Lambda^{(2)}$, the center wavelength of the first band as represented by λ_1 , the center wavelength of the first band
10 as represented by λ_2 , and scaling parameters σ_γ and σ_φ , as follows:

$$\begin{aligned} \left(\mathbf{R}_j^\gamma\right)^{-1} &= \left[\begin{array}{c|c} \frac{1}{\sigma_\gamma^2} \mathbf{W} & \mathbf{O}_{n \times n} \\ \hline \mathbf{O}_{n \times n} & \frac{1}{\sigma_\gamma^2} \mathbf{W} \end{array} \right], \\ \left(\mathbf{R}_j^\varphi\right)^{-1} &= \left[\begin{array}{c|c} \frac{1}{\sigma_\varphi^2} \mathbf{W}^{(1)} & \mathbf{O}_{n \times n} \\ \hline \mathbf{O}_{n \times n} & \frac{1}{\sigma_\varphi^2} \mathbf{W}^{(2)} \end{array} \right], \end{aligned}$$

$$\text{where } \mathbf{W}^{(1)} = \frac{1}{\lambda_1^2} \Lambda^{(1)} \mathbf{W} \Lambda^{(1)}, \quad \mathbf{W}^{(2)} = \frac{1}{\lambda_2^2} \Lambda^{(2)} \mathbf{W} \Lambda^{(2)},$$

15 wherein step (h) of generating matrix \mathbf{T}_j comprises the steps of:

generating a scalar b in a form equivalent to: $b = \frac{\sigma_r^2 \lambda_1^2 \lambda_2^2}{2\sigma_\phi^2 \lambda_1^2 \lambda_2^2 + \sigma_r^2 \lambda_1^2 + \sigma_r^2 \lambda_2^2}$,

where λ_1 is the wavelength of a first group of satellite signals and λ_2 is the wavelength of a second group of satellite signals,

generating a matrix $\tilde{\mathbf{H}}$ in a form equivalent to $\tilde{\mathbf{H}} = \mathbf{W}^{1/2} \mathbf{H}_j^r$,

20

generating a Householder matrix $\mathbf{S}_{\mathbf{HH}}$ for matrix $\tilde{\mathbf{H}}$, and

generating matrix \mathbf{T}_j in a form equivalent to: $\mathbf{T}_j = \frac{1}{\sigma_\phi} \begin{bmatrix} \mathbf{A11} & | & \mathbf{O}_{n \times n} \\ \hline \mathbf{A21} & | & \mathbf{A22} \end{bmatrix}$,

where sub-matrixes $\mathbf{A11}$, $\mathbf{A21}$, and $\mathbf{A22}$ are as follows:

$$\mathbf{A11} = \left(\mathbf{W}^{(1)} \right)^{\frac{1}{2}} \mathbf{S}_{\mathbf{HH}} \begin{bmatrix} \sqrt{1-b/\lambda_1^2} \mathbf{I}_4 & | & \mathbf{O}_{4 \times (n-4)} \\ \hline \mathbf{O}_{(n-4) \times 4} & | & \mathbf{I}_{(n-4)} \end{bmatrix},$$

$$\mathbf{A21} = \left(\mathbf{W}^{(2)} \right)^{\frac{1}{2}} \mathbf{S}_{\mathbf{HH}} \begin{bmatrix} -\frac{b}{\lambda_1 \lambda_2 \sqrt{1-b/\lambda_1^2}} \mathbf{I}_4 & | & \mathbf{O}_{4 \times (n-4)} \\ \hline \mathbf{O}_{(n-4) \times 4} & | & \mathbf{O}_{(n-4) \times (n-4)} \end{bmatrix}, \text{ and}$$

$$\mathbf{A22} = \left(\mathbf{W}^{(2)} \right)^{\frac{1}{2}} \mathbf{S}_{\mathbf{HH}} \begin{bmatrix} \sqrt{\frac{1-b/\lambda_1^2 - b/\lambda_2^2}{1-b/\lambda_1^2}} \mathbf{I}_4 & | & \mathbf{O}_{4 \times (n-4)} \\ \hline \mathbf{O}_{(n-4) \times 4} & | & \mathbf{I}_{(n-4)} \end{bmatrix}.$$

25

33. A computer program product for directing a data processor to estimate a set of floating ambiguities associated with a set of phase measurements of a plurality n of satellite carrier signals made by a first navigation receiver (B) and a second navigation receiver (R) separated by a distance, wherein a baseline vector (x^o, y^o, z^o) relates the position of the second receiver to the first receiver, each satellite carrier signal being transmitted by a satellite and having a wavelength, wherein each receiver has a time clock

5

for referencing its measurements and wherein any difference between the time clocks may be represented by an offset, the process receiving, for a plurality of two or more time moments j , the following inputs:

- 10 a vector γ_j^B representative of a plurality of pseudo-ranges measured by the first navigation receiver (B) and corresponding to the plurality of satellite carrier signals,
- a vector γ_j^R representative of a plurality of pseudo-ranges measured by the second navigation receiver (R) and corresponding to the plurality of satellite carrier signals,
- a vector D_j^B representative of a plurality of estimated distances between the
- 15 satellites and the first navigation receiver (B),
- a vector D_j^R representative of a plurality of estimated distances between the satellites and the second navigation receiver (R),
- a vector ϕ_j^B representative of a plurality of full phase measurements of the satellite carrier signals measured by the first navigation receiver (B),
- 20 a vector ϕ_j^R representative of a plurality of full phase measurements of the satellite carrier signals measured by the second navigation receiver (R),
- a geometric Jacobian matrix H_j^γ whose matrix elements are representative of the changes in the distances between the satellites and one of the receivers that would be caused by changes in that receiver's position and time clock offset, the computer program
- 25 product comprising:
 - a computer-readable medium;
 - a first set of instructions embodied on the computer-readable medium which directs the data processor to generate, for each time moment j , a vector $\Delta\gamma_j$ of a plurality of range residuals of pseudo-range measurements made by the first and second navigation
 - 30 receivers in the form of: $\Delta\gamma_j = (\gamma_j^R - \gamma_j^B) - (D_j^R - D_j^B)$;
 - a second set of instructions embodied on the computer-readable medium which directs the data processor to generate, for each time moment j , a vector $\Delta\phi_j$ of a plurality of phase residuals of full phase measurements made by the first and second navigation
 - receivers in the form of: $\Delta\phi_j = (\phi_j^R - \phi_j^B) - \Lambda^{-1} \cdot (D_j^R - D_j^B)$, where Λ^{-1} is a diagonal
 - 35 matrix comprising the inverse wavelengths of the satellites;

a third set of instructions embodied on the computer-readable medium which directs the data processor to generate an LU-factorization of a matrix \mathbf{M} or a matrix inverse of matrix \mathbf{M} , the matrix \mathbf{M} being a function of at least $\mathbf{\Lambda}^{-1}$ and \mathbf{H}_k^γ , for index k of \mathbf{H}_k^γ covering at least two of the time moments j ; and

40 a fourth set of instructions embodied on the computer-readable medium which directs the data processor to generate a vector \mathbf{N} of estimated floating ambiguities as a function of at least the set of range residuals $\Delta\gamma_k$, the set of phase residuals $\Delta\phi_k$, and the LU-factorization of matrix \mathbf{M} or the matrix inverse of matrix \mathbf{M} .

34. The computer program product of Claim 33 wherein the third set of instructions directs the data processor to generate matrix \mathbf{M} in a form equivalent to the summation

$$\left[\sum_{k=1}^j (\mathbf{G}^T \mathbf{P}_k \mathbf{G}) \right],$$

5 where:

- the matrix \mathbf{G} has $2n$ rows, n columns, an upper sub-matrix, and a lower sub-matrix, one of the sub-matrices comprises an $n \times n$ zero matrix and the other sub-matrix comprising an $n \times n$ identity matrix,
- the matrix \mathbf{G}^T comprises the transpose matrix of matrix \mathbf{G} , and

10 • the matrix \mathbf{P}_k has $2n$ rows, $2n$ columns, and a form which comprises a matrix

equivalent to $\mathbf{P}_k = \mathbf{R}_k^{-1} - \mathbf{R}_k^{-1} \mathbf{Q}_k \left(\mathbf{Q}_k^T \mathbf{R}_k^{-1} \mathbf{Q}_k + q\mathbf{S}_k \right)^{-1} \mathbf{Q}_k^T \mathbf{R}_k^{-1}$, where the matrix \mathbf{R}_k is a weighting matrix, where the matrix \mathbf{R}_k^{-1} comprises an inverse of matrix \mathbf{R}_k , where the matrix \mathbf{Q}_k has an upper sub-matrix and a lower sub-matrix, one of the sub-matrices of \mathbf{Q}_k comprising matrix \mathbf{H}_k^γ and the other of the sub-matrices of \mathbf{Q}_k comprising the matrix product $\mathbf{\Lambda}^{-1} \mathbf{H}_k^\gamma$, and wherein the matrix \mathbf{Q}_k^T comprises the transpose of matrix \mathbf{Q}_k , and where the quantity $q\mathbf{S}_k$ is a zero matrix when the distance between the first and second navigation receivers is unconstrained and where q may be a non-zero weighting parameter and \mathbf{S}_k may be a non-zero matrix when the distance between the first and second navigation receivers is constrained.

20

35. The computer program product of Claim 34 wherein the fourth set of instructions directs the data processor to generate matrix \mathbf{N} in a form equivalent to:

$$\mathbf{N} = \mathbf{M}^{-1} \times \left[\sum_{k=1}^j (\mathbf{G}^T \mathbf{P}_k \boldsymbol{\mu}_k + q \mathbf{g}_k) \right],$$

where the matrix \mathbf{M}^{-1} comprises an inverse of matrix of matrix \mathbf{M} , where the vector $\boldsymbol{\mu}_k$ comprises the vector $[\Delta\gamma_k, \Delta\varphi_k]^T$, and where the quantity $q\mathbf{g}_k$ is a zero vector when the distance between the first and second navigation receivers is unconstrained and where q may be non-zero and \mathbf{g}_k may be a non-zero vector when the distance between the first and second navigation receivers is constrained.

36. The computer program product of Claim 35 wherein the distance between the first and second navigation receivers is constrained to a distance L_{RB} , wherein the third set of instructions directs the data processor to generate matrix \mathbf{S}_k for the k -th time moment in a form equivalent to:

$$\mathbf{S}_k = \left(1 - \frac{L_{RB}}{\|\mathbf{r}_k\|} \right) \begin{pmatrix} \mathbf{I}_3 & \mathbf{O}_{3 \times 1} \\ \mathbf{O}_{1 \times 3} & 0 \end{pmatrix} + \frac{L_{RB}}{\|\mathbf{r}_k\|} \mathbf{r}_k \mathbf{r}_k^T$$

where \mathbf{r}_k is a vector comprising estimates of the three coordinates of the baseline vector at the k -th time moment, and a zero as fourth component, where \mathbf{r}_k^T is the vector transpose of \mathbf{r}_k , where \mathbf{I}_3 is the 3-by-3 identity matrix, where $\mathbf{O}_{1 \times 3}$ is a row vector of three zeros, and where $\mathbf{O}_{3 \times 1}$ is a column vector of three zeros; and

wherein step (d) generates vector \mathbf{g}_k for the k -th time moment in a form equivalent to:

$$\mathbf{g}_k = \mathbf{G}^T \mathbf{R}_k^{-1} \mathbf{Q}_k (\mathbf{Q}_k^T \mathbf{R}_k^{-1} \mathbf{Q}_k + q \mathbf{S}_k)^{-1} \mathbf{h}_k,$$

where:

$$\mathbf{h}_k = \left(1 - \frac{L_{RB}}{\|\mathbf{r}_k\|} \right) \mathbf{r}_k.$$

37. An apparatus for estimating a set of floating ambiguities associated with a set of phase measurements of a plurality n of satellite carrier signals made by a first navigation receiver (B) and a second navigation receiver (R) separated by a distance,

wherein a baseline vector (x^o, y^o, z^o) relates the position of the second receiver to the first receiver, each satellite carrier signal being transmitted by a satellite and having a wavelength, wherein each receiver has a time clock for referencing its measurements and wherein any difference between the time clocks may be represented by an offset, said apparatus receiving, for a plurality of two or more time moments j , the following inputs:

a vector γ_j^B representative of a plurality of pseudo-ranges measured by the first navigation receiver (B) and corresponding to the plurality of satellite carrier signals,

a vector γ_j^R representative of a plurality of pseudo-ranges measured by the second navigation receiver (R) and corresponding to the plurality of satellite carrier signals,

a vector D_j^B representative of a plurality of estimated distances between the satellites and the first navigation receiver (B),

a vector D_j^R representative of a plurality of estimated distances between the satellites and the second navigation receiver (R),

a vector φ_j^B representative of a plurality of full phase measurements of the satellite carrier signals measured by the first navigation receiver (B),

a vector φ_j^R representative of a plurality of full phase measurements of the satellite carrier signals measured by the second navigation receiver (R),

a geometric Jacobian matrix H_j^γ whose matrix elements are representative of the changes in the distances between the satellites and one of the receivers that would be caused by changes in that receiver's position and time clock offset,

said apparatus comprising:

(a) means for generating, for each time moment j , a vector $\Delta\gamma_j$ of a plurality of range residuals of pseudo-range measurements made by the first and second navigation receivers in the form of: $\Delta\gamma_j = (\gamma_j^R - \gamma_j^B) - (D_j^R - D_j^B)$;

(b) means for generating, for each time moment j , a vector $\Delta\varphi_j$ of a plurality of phase residuals of full phase measurements made by the first and second navigation

receivers in the form of: $\Delta\varphi_j = (\varphi_j^R - \varphi_j^B) - \Lambda^{-1} \cdot (D_j^R - D_j^B)$, where Λ^{-1} is a diagonal matrix comprising the inverse wavelengths of the satellites;

(c) means for generating, for time moment $j = 1$, an LU-factorization of a matrix \mathbf{M}_1 or a matrix inverse of matrix \mathbf{M}_1 , the matrix \mathbf{M}_1 being a function of at least $\mathbf{\Lambda}^{-1}$ and \mathbf{H}_1^γ ;

35 (d) means for generating, for time moment $j = 1$, a vector \mathbf{N}_1 as a function of at least $\Delta\gamma_1$, $\Delta\varphi_1$, and the LU-factorization of matrix \mathbf{M}_1 or the matrix inverse of matrix \mathbf{M}_1 ;

(e) means for generating, for an additional time moment $j \neq 1$, an LU-factorization of a matrix \mathbf{M}_j or a matrix inverse of matrix \mathbf{M}_j , the matrix \mathbf{M}_j being a function of at least
40 $\mathbf{\Lambda}^{-1}$ and \mathbf{H}_j^γ ; and

(f) means for generating, for an additional time moment $j \neq 1$, a vector \mathbf{N}_j as a function of at least $\Delta\gamma_j$, $\Delta\varphi_j$, and the LU-factorization or matrix \mathbf{M}_j or the matrix inverse of matrix \mathbf{M}_j , the vector \mathbf{N}_j having estimates of the floating ambiguities.

38. The apparatus of Claim 37 wherein means (c) comprises means for generating an LU-factorization for a matrix comprising a form equivalent to $(\mathbf{G}^T \mathbf{P}_1 \mathbf{G})$, where:

- the matrix \mathbf{G} has $2n$ rows, n columns, an upper sub-matrix, and a lower sub-matrix, one of the sub-matrices comprises an $n \times n$ zero matrix and the other sub-matrix
5 comprising an $n \times n$ identity matrix,

- the matrix \mathbf{G}^T comprises the transpose matrix of matrix \mathbf{G} , and
- the matrix \mathbf{P}_1 has $2n$ rows, $2n$ columns, and a form which comprises a matrix

equivalent to $\left(\mathbf{R}_1^{-1} - \mathbf{R}_1^{-1} \mathbf{Q}_1 (\mathbf{Q}_1^T \mathbf{R}_1^{-1} \mathbf{Q}_1 + q \mathbf{S}_1) \right)^{-1} \mathbf{Q}_1^T \mathbf{R}_1^{-1}$ where the matrix \mathbf{R}_1

10 is a weighting matrix, where the matrix \mathbf{R}_1^{-1} comprises an inverse of matrix \mathbf{R}_1 , where the matrix \mathbf{Q}_1 has an upper sub-matrix and a lower sub-matrix, one of the sub-matrices of \mathbf{Q}_1 comprising matrix \mathbf{H}_1^γ and the other of the sub-matrices of \mathbf{Q}_1

comprising the matrix product $\mathbf{\Lambda}^{-1} \mathbf{H}_1^\gamma$, and wherein the matrix \mathbf{Q}_1^T comprises the transpose of matrix \mathbf{Q}_1 , and where the quantity $q \mathbf{S}_k$ is a zero matrix when the distance between the first and second navigation receivers is unconstrained and where q may
15 be a non-zero weighting parameter and \mathbf{S}_k may be a non-zero matrix when the distance between the first and second navigation receivers is constrained.

39. The apparatus of Claim 38 wherein means (d) comprises means for generating vector \mathbf{N}_1 to comprise a vector having a form equivalent to $\mathbf{M}_1^{-1}(\mathbf{G}^T \mathbf{P}_1 \boldsymbol{\mu}_1 + q\mathbf{g}_1)$, where the matrix \mathbf{M}_1^{-1} comprises an inverse of matrix of matrix \mathbf{M}_1 , and where the vector $\boldsymbol{\mu}_1$ comprises the vector $[\Delta\gamma_1, \Delta\varphi_1]^T$, and where the quantity $q\mathbf{g}_k$ is a zero vector when the distance between the first and second navigation receivers is unconstrained and where q may be non-zero and \mathbf{g}_k may be a non-zero vector when the distance between the first and second navigation receivers is constrained.

40. The apparatus of Claim 37 wherein means (e) comprises means for generating an LU-factorization for a matrix comprising a form equivalent to

$$\mathbf{M}_j = \mathbf{M}_{j-1} + \mathbf{G}^T \mathbf{P}_j \mathbf{G}, \text{ where:}$$

- \mathbf{M}_{j-1} comprises the matrix \mathbf{M}_1 of generated by means (c) when $j = 2$ and comprises the matrix \mathbf{M}_j generated by means (e) for the $j-1$ time moment when $j > 2$,
- the matrix \mathbf{G} has $2n$ rows, n columns, an upper sub-matrix, and a lower sub-matrix, one of the sub-matrices comprising an $n \times n$ zero matrix and the other sub-matrix comprising an $n \times n$ identity matrix,
- the matrix \mathbf{G}^T comprises the transpose matrix of matrix \mathbf{G} , and
- the matrix \mathbf{P}_j has $2n$ rows, $2n$ columns, and a form which comprise a matrix

equivalent to $\left(\mathbf{R}_j^{-1} - \mathbf{R}_j^{-1} \mathbf{Q}_j (\mathbf{Q}_j^T \mathbf{R}_j^{-1} \mathbf{Q}_j + q\mathbf{S}_j)^{-1} \mathbf{Q}_j^T \mathbf{R}_j^{-1} \right)$ where the matrix

\mathbf{R}_j is a weighting matrix, where the matrix \mathbf{R}_j^{-1} comprises an inverse of matrix \mathbf{R}_j , where the matrix \mathbf{Q}_j has an upper sub-matrix and a lower sub-matrix, one of the sub-matrices of \mathbf{Q}_j comprising matrix \mathbf{H}_j^T and the other of the sub-matrices of \mathbf{Q}_j

comprising the matrix product $\mathbf{A}^{-1} \mathbf{H}_j^T$, and wherein the matrix \mathbf{Q}_j^T comprises the transpose of matrix \mathbf{Q}_j , and where the quantity $q\mathbf{S}_j$ is a zero matrix when the distance between the first and second navigation receivers is unconstrained and where q may be a non-zero weighting parameter and \mathbf{S}_j may be a non-zero matrix when the distance between the first and second navigation receivers is constrained.

41. The apparatus of Claim 40 wherein means (f) comprises means for generating vector \mathbf{N}_j to comprise a vector having a form equivalent to

$\mathbf{N}_{j-1} + \mathbf{M}_j^{-1} \left[\mathbf{G}^T \mathbf{P}_j (\boldsymbol{\mu}_j - \mathbf{G} \mathbf{N}_{j-1}) + q \mathbf{g}_j \right]$, where the matrix \mathbf{M}_j^{-1} comprises an inverse of matrix of matrix \mathbf{M}_j , where the vector $\boldsymbol{\mu}_j$ comprises the vector $[\Delta \gamma_j, \Delta \varphi_j]^T$, and where the vector \mathbf{N}_{j-1} comprises the vector \mathbf{N}_1 generated by means (d) when $j = 2$ and comprises the vector \mathbf{N}_{j-1} generated by means (f) for the $j-1$ time moment when $j > 2$, and where the quantity $q \mathbf{g}_j$ is a zero vector when the distance between the first and second navigation receivers is unconstrained and where q may be non-zero and \mathbf{g}_j may be a non-zero vector when the distance between the first and second navigation receivers is constrained.

42. The apparatus of claim 39 wherein the distance between the first and second navigation receivers is constrained to a distance L_{RB} , wherein means (c) comprises means for generating matrix \mathbf{S}_1 in a form equivalent to:

$$\mathbf{S}_1 = \left(1 - \frac{L_{RB}}{\|\mathbf{r}_1\|} \right) \begin{pmatrix} \mathbf{I}_3 & \mathbf{O}_{3 \times 1} \\ \mathbf{O}_{1 \times 3} & 0 \end{pmatrix} + \frac{L_{RB}}{\|\mathbf{r}_1\|} \mathbf{r}_1 \mathbf{r}_1^T$$

where \mathbf{r}_1 is a vector comprising estimates of the three coordinates of the baseline vector for the time moment $j=1$ and a zero as fourth component, where \mathbf{r}_1^T is the vector transpose of \mathbf{r}_1 , where \mathbf{I}_3 is the 3-by-3 identity matrix, where $\mathbf{O}_{1 \times 3}$ is a row vector of three zeros, and where $\mathbf{O}_{3 \times 1}$ is a column vector of three zeros; and

wherein means (d) comprises means for generating vector \mathbf{g}_1 for the time moment $j=1$ in a form equivalent to:

$$\mathbf{g}_1 = \mathbf{G}^T \mathbf{R}_1^{-1} \mathbf{Q}_1 (\mathbf{Q}_1^T \mathbf{R}_1^{-1} \mathbf{Q}_1 + q \mathbf{S}_1)^{-1} \mathbf{h}_1,$$

where:

$$\mathbf{h}_1 = \left(1 - \frac{L_{RB}}{\|\mathbf{r}_1\|} \right) \mathbf{r}_1.$$

43. The apparatus of Claim 41 wherein the distance between the first and second navigation receivers is constrained to a distance L_{RB} , wherein means (e) comprises means for generating matrix \mathbf{S}_j in a form equivalent to:

$$\mathbf{S}_j = \left(1 - \frac{L_{RB}}{\|\mathbf{r}_j\|} \right) \begin{pmatrix} \mathbf{I}_3 & \mathbf{O}_{3 \times 1} \\ \mathbf{O}_{1 \times 3} & 0 \end{pmatrix} + \frac{L_{RB}}{\|\mathbf{r}_j\|} \mathbf{r}_j \mathbf{r}_j^T$$

- 5 where \mathbf{r}_j is a vector comprising estimates of the three coordinates of the baseline vector for the j-th time moment and a zero as fourth vector component, where \mathbf{r}_j^T is the vector transpose of \mathbf{r}_j , where \mathbf{I}_3 is the 3-by-3 identity matrix, where $\mathbf{O}_{1 \times 3}$ is a row vector of three zeros, and where $\mathbf{O}_{3 \times 1}$ is a column vector of three zeros; and

- 10 wherein means (f) comprises means for generating vector \mathbf{g}_j for the j-th time moment in a form equivalent to:

$$\mathbf{g}_j = \mathbf{G}^T \mathbf{R}_j^{-1} \mathbf{Q}_j (\mathbf{Q}_j^T \mathbf{R}_j^{-1} \mathbf{Q}_j + q \mathbf{S}_j)^{-1} \mathbf{h}_j,$$

where:

$$\mathbf{h}_j = \left(1 - \frac{L_{RB}}{\|\mathbf{r}_j\|} \right) \mathbf{r}_j.$$

44. The apparatus of Claim 40 wherein means (e) for generating the LU-factorization of \mathbf{M}_j comprises:

- (g) means for generating an LU-factorization of matrix \mathbf{M}_{j-1} in a form equivalent to $\mathbf{L}_{j-1} \mathbf{L}_{j-1}^T$ wherein \mathbf{L}_{j-1} is a low-triangular matrix and \mathbf{L}_{j-1}^T is the transpose of \mathbf{L}_{j-1} ;
- 5 (h) means for generating a factorization of $\mathbf{G}^T \mathbf{P}_j \mathbf{G}$ in a form equivalent to $\mathbf{T}_j \mathbf{T}_j^T = \mathbf{G}^T \mathbf{P}_j \mathbf{G}$, where \mathbf{T}_j^T is the transpose of \mathbf{T}_j ; and
- (i) means for generating an LU-factorization of matrix \mathbf{M}_j in a form equivalent to $\mathbf{L}_j \mathbf{L}_j^T$ from a plurality n of rank-one modifications of matrix \mathbf{L}_{j-1} , each rank-one
- 10 modification being based on a respective column of matrix \mathbf{T}_j , where n is the number of rows in matrix \mathbf{M}_j .

45. The apparatus of Claim 44 wherein means (h) generates matrix \mathbf{T}_j from a Cholesky factorization of $\mathbf{G}^T \mathbf{P}_j \mathbf{G}$.

46. The apparatus of Claim 44 wherein weighting matrix \mathbf{R}_j has a form equivalent to:

$$\mathbf{R}_j = \begin{bmatrix} \mathbf{R}_j^\gamma & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_j^\varphi \end{bmatrix}, \text{ where } \mathbf{R}^\gamma \text{ and } \mathbf{R}^\varphi \text{ are weighting matrices;}$$

wherein \mathbf{R}^γ and \mathbf{R}^φ are related to a common weighting matrix \mathbf{W} and scaling parameters σ_γ and σ_φ as follows $(\mathbf{R}^\gamma)^{-1} = \frac{1}{\sigma_\gamma^2} \mathbf{W}$, and $(\mathbf{R}^\varphi)^{-1} = \frac{1}{\sigma_\varphi^2} \mathbf{W}$,

wherein means (h) of generating matrix \mathbf{T}_j comprises:

means for generating a scalar b in a form equivalent to: $b = \frac{\sigma_\gamma^2 \lambda_{GPS}^2}{\sigma_\gamma^2 + \lambda_{GPS}^2 \sigma_\varphi^2}$, where

λ_{GPS}^2 is the wavelength of the satellite signals,

means for generating a matrix $\tilde{\mathbf{H}}$ in a form equivalent to $\tilde{\mathbf{H}} = \mathbf{W}^{1/2} \mathbf{H}_j^\gamma$,

means for generating a Householder matrix $\mathbf{S}_{\mathbf{H}\mathbf{H}}$ for matrix $\tilde{\mathbf{H}}$, and

means for generating matrix \mathbf{T}_j in a form equivalent to:

$$\mathbf{T}_j = \frac{1}{\sigma_\varphi} \mathbf{W}^{1/2} \mathbf{S}_{\mathbf{H}\mathbf{H}} \begin{bmatrix} \sqrt{\left(1 - \frac{b}{\lambda_{GPS}^2}\right)} \mathbf{I}_4 & | & \mathbf{0}_{4 \times (n-4)} \\ \text{---} & | & \text{---} \\ \mathbf{0}_{(n-4) \times 4} & | & \mathbf{I}_{(n-4) \times (n-4)} \end{bmatrix}.$$

47. The method of Claim 44 wherein weighting matrix \mathbf{R}_j is applied to a case where there is a first group of satellite signals having carrier signals in a first wavelength band and a second group of satellite signals having a carrier signals in a second wavelength band, the weighting frequency having a form equivalent to:

$$\mathbf{R}_j = \begin{bmatrix} \mathbf{R}_j^\gamma & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_j^\varphi \end{bmatrix}, \text{ where } \mathbf{R}^\gamma \text{ and } \mathbf{R}^\varphi \text{ are weighting matrices;}$$

wherein \mathbf{R}^γ and \mathbf{R}^φ are related to a common weighting matrix \mathbf{W} , the carrier wavelengths of the first group of signals as represented by matrix $\Lambda^{(1)}$, the carrier

wavelengths of the second group of signals as represented by matrix $\Lambda^{(2)}$, the center wavelength of the first band as represented by λ_1 , the center wavelength of the first band

10 as represented by λ_2 , and scaling parameters σ_γ and σ_φ , as follows:

$$\begin{aligned} \left(\mathbf{R}_j^\gamma\right)^{-1} &= \left[\begin{array}{c|c} \frac{1}{\sigma_\gamma^2} \mathbf{W} & \mathbf{O}_{n \times n} \\ \hline \mathbf{O}_{n \times n} & \frac{1}{\sigma_\gamma^2} \mathbf{W} \end{array} \right], \\ \left(\mathbf{R}_j^\varphi\right)^{-1} &= \left[\begin{array}{c|c} \frac{1}{\sigma_\varphi^2} \mathbf{W}^{(1)} & \mathbf{O}_{n \times n} \\ \hline \mathbf{O}_{n \times n} & \frac{1}{\sigma_\varphi^2} \mathbf{W}^{(2)} \end{array} \right], \end{aligned}$$

$$\text{where } \mathbf{W}^{(1)} = \frac{1}{\lambda_1^2} \Lambda^{(1)} \mathbf{W} \Lambda^{(1)}, \quad \mathbf{W}^{(2)} = \frac{1}{\lambda_2^2} \Lambda^{(2)} \mathbf{W} \Lambda^{(2)},$$

15 wherein step (h) of generating matrix \mathbf{T}_j comprises the steps of:

$$\text{generating a scalar } b \text{ in a form equivalent to: } b = \frac{\sigma_\gamma^2 \lambda_1^2 \lambda_2^2}{2\sigma_\varphi^2 \lambda_1^2 \lambda_2^2 + \sigma_\gamma^2 \lambda_1^2 + \sigma_\gamma^2 \lambda_2^2},$$

where λ_1 is the wavelength of a first group of satellite signals and λ_2 is the wavelength of a second group of satellite signals,

$$\text{generating a matrix } \tilde{\mathbf{H}} \text{ in a form equivalent to } \tilde{\mathbf{H}} = \mathbf{W}^{1/2} \mathbf{H}_j^\gamma,$$

20 generating a Householder matrix $\mathbf{S}_{\mathbf{HH}}$ for matrix $\tilde{\mathbf{H}}$, and

$$\text{generating matrix } \mathbf{T}_j \text{ in a form equivalent to: } \mathbf{T}_j = \frac{1}{\sigma_\varphi} \left[\begin{array}{c|c} \mathbf{A11} & \mathbf{O}_{n \times n} \\ \hline \mathbf{A21} & \mathbf{A22} \end{array} \right],$$

where sub-matrixces **A11**, **A21**, and **A22** are as follows:

$$\begin{aligned}
\mathbf{A11} &= \left(\mathbf{W}^{(1)}\right)^{\frac{1}{2}} \mathbf{S}_{\mathbf{HH}} \left[\begin{array}{c|c} \sqrt{1-b/\lambda_1^2} \mathbf{I}_4 & \mathbf{O}_{4 \times (n-4)} \\ \hline \mathbf{O}_{(n-4) \times 4} & \mathbf{I}_{(n-4)} \end{array} \right], \\
\mathbf{A21} &= \left(\mathbf{W}^{(2)}\right)^{\frac{1}{2}} \mathbf{S}_{\mathbf{HH}} \left[\begin{array}{c|c} -\frac{b}{\lambda_1 \lambda_2 \sqrt{1-b/\lambda_1^2}} \mathbf{I}_4 & \mathbf{O}_{4 \times (n-4)} \\ \hline \mathbf{O}_{(n-4) \times 4} & \mathbf{O}_{(n-4) \times (n-4)} \end{array} \right], \text{ and} \\
\mathbf{A22} &= \left(\mathbf{W}^{(2)}\right)^{\frac{1}{2}} \mathbf{S}_{\mathbf{HH}} \left[\begin{array}{c|c} \sqrt{\frac{1-b/\lambda_1^2 - b/\lambda_2^2}{1-b/\lambda_1^2}} \mathbf{I}_4 & \mathbf{O}_{4 \times (n-4)} \\ \hline \mathbf{O}_{(n-4) \times 4} & \mathbf{I}_{(n-4)} \end{array} \right].
\end{aligned}$$

25

48. An apparatus for estimating a set of floating ambiguities associated with a set of phase measurements of a plurality n of satellite carrier signals made by a first navigation receiver (B) and a second navigation receiver (R) separated by a distance, wherein a baseline vector (x^o, y^o, z^o) relates the position of the second receiver to the first receiver, each satellite carrier signal being transmitted by a satellite and having a wavelength, wherein each receiver has a time clock for referencing its measurements and wherein any difference between the time clocks may be represented by an offset, said apparatus receiving, for a plurality of two or more time moments j , the following inputs:

- a vector γ_j^B representative of a plurality of pseudo-ranges measured by the first navigation receiver (B) and corresponding to the plurality of satellite carrier signals,
- a vector γ_j^R representative of a plurality of pseudo-ranges measured by the second navigation receiver (R) and corresponding to the plurality of satellite carrier signals,
- a vector D_j^B representative of a plurality of estimated distances between the satellites and the first navigation receiver (B),
- a vector D_j^R representative of a plurality of estimated distances between the satellites and the second navigation receiver (R),

a vector $\boldsymbol{\varphi}_j^B$ representative of a plurality of full phase measurements of the satellite carrier signals measured by the first navigation receiver (B),

20 a vector $\boldsymbol{\varphi}_j^R$ representative of a plurality of full phase measurements of the satellite carrier signals measured by the second navigation receiver (R),

a geometric Jacobian matrix \mathbf{H}_j^γ whose matrix elements are representative of the changes in the distances between the satellites and one of the receivers that would be caused by changes in that receiver's position and time clock offset,

said apparatus comprising:

25 (a) means for generating, for each time moment j , a vector $\Delta\boldsymbol{\gamma}_j$ of a plurality of range residuals of pseudo-range measurements made by the first and second navigation receivers in the form of: $\Delta\boldsymbol{\gamma}_j = (\boldsymbol{\gamma}_j^R - \boldsymbol{\gamma}_j^B) - (\mathbf{D}_j^R - \mathbf{D}_j^B)$, said means generating a set of range residuals $\Delta\boldsymbol{\gamma}_k$, $k=1, \dots, j$;

30 (b) means for generating, for each time moment j , a vector $\Delta\boldsymbol{\varphi}_j$ of a plurality of phase residuals of full phase measurements made by the first and second navigation receivers in the form of: $\Delta\boldsymbol{\varphi}_j = (\boldsymbol{\varphi}_j^R - \boldsymbol{\varphi}_j^B) - \boldsymbol{\Lambda}^{-1} \cdot (\mathbf{D}_j^R - \mathbf{D}_j^B)$, where $\boldsymbol{\Lambda}^{-1}$ is a diagonal matrix comprising the inverse wavelengths of the satellites, said means generating a set of phase residuals $\Delta\boldsymbol{\varphi}_k$, $k=1, \dots, j$;

35 (c) means for generating an LU-factorization of a matrix \mathbf{M} or a matrix inverse of matrix \mathbf{M} , the matrix \mathbf{M} being a function of at least $\boldsymbol{\Lambda}^{-1}$ and \mathbf{H}_k^γ , for index k of \mathbf{H}_k^γ covering at least two of the time moments j ;

(d) means for generating a vector \mathbf{N} of estimated floating ambiguities as a function of at least the set of range residuals $\Delta\boldsymbol{\gamma}_k$, the set of phase residuals $\Delta\boldsymbol{\varphi}_k$, and the LU-factorization of matrix \mathbf{M} or the matrix inverse of matrix \mathbf{M} .

49. The apparatus of Claim 48 wherein means (c) comprises means for generating matrix \mathbf{M} in a form equivalent to the summation
$$\left[\sum_{k=1}^j (\mathbf{G}^T \mathbf{P}_k \mathbf{G}) \right],$$

where:

- the matrix \mathbf{G} has $2n$ rows, n columns, an upper sub-matrix, and a lower sub-matrix, one of the sub-matrices comprises an $n \times n$ zero matrix and the other sub-matrix comprising an $n \times n$ identity matrix,
- the matrix \mathbf{G}^T comprises the transpose matrix of matrix \mathbf{G} , and
- the matrix \mathbf{P}_k has $2n$ rows, $2n$ columns, and a form which comprises a matrix equivalent to $\mathbf{P}_k = \mathbf{R}_k^{-1} - \mathbf{R}_k^{-1} \mathbf{Q}_k (\mathbf{Q}_k^T \mathbf{R}_k^{-1} \mathbf{Q}_k + q\mathbf{S}_k)^{-1} \mathbf{Q}_k^T \mathbf{R}_k^{-1}$, where the matrix \mathbf{R}_k is a weighting matrix, where the matrix \mathbf{R}_k^{-1} comprises an inverse of matrix \mathbf{R}_k , where the matrix \mathbf{Q}_k has an upper sub-matrix and a lower sub-matrix, one of the sub-matrices of \mathbf{Q}_k comprising matrix \mathbf{H}_k^γ and the other of the sub-matrices of \mathbf{Q}_k comprising the matrix product $\mathbf{\Lambda}^{-1} \mathbf{H}_k^\gamma$, and wherein the matrix \mathbf{Q}_k^T comprises the transpose of matrix \mathbf{Q}_k , and where the quantity $q\mathbf{S}_k$ is a zero matrix when the distance between the first and second navigation receivers is unconstrained and where q may be a non-zero weighting parameter and \mathbf{S}_k may be a non-zero matrix when the distance between the first and second navigation receivers is constrained.

50. The apparatus of Claim 49 wherein means (d) comprises means for generating matrix \mathbf{N} in a form equivalent to:

$$\mathbf{N} = \mathbf{M}^{-1} \times \left[\sum_{k=1}^j (\mathbf{G}^T \mathbf{P}_k \boldsymbol{\mu}_k + q\mathbf{g}_k) \right],$$

- where the matrix \mathbf{M}^{-1} comprises an inverse of matrix of matrix \mathbf{M} , where the vector $\boldsymbol{\mu}_k$ comprises the vector $[\Delta\gamma_k, \Delta\varphi_k]^T$, and where the quantity $q\mathbf{g}_k$ is a zero vector when the distance between the first and second navigation receivers is unconstrained and where q may be non-zero and \mathbf{g}_k may be a non-zero vector when the distance between the first and second navigation receivers is constrained.

51. The apparatus of claim 50 wherein the distance between the first and second navigation receivers is constrained to a distance L_{RB} , wherein means (c) comprises means for generating matrix \mathbf{S}_k for the k -th time moment in a form equivalent to:

$$\mathbf{S}_k = \left(1 - \frac{L_{RB}}{\|\mathbf{r}_k\|} \right) \begin{pmatrix} \mathbf{I}_3 & \mathbf{O}_{3 \times 1} \\ \mathbf{O}_{1 \times 3} & 0 \end{pmatrix} + \frac{L_{RB}}{\|\mathbf{r}_k\|} \mathbf{r}_k \mathbf{r}_k^T$$

5 where \mathbf{r}_k is a vector comprising estimates of the three coordinates of the baseline vector at the k-th time moment, and a zero as fourth component, where \mathbf{r}_k^T is the vector transpose of \mathbf{r}_k , where \mathbf{I}_3 is the 3-by-3 identity matrix, where $\mathbf{O}_{1 \times 3}$ is a row vector of three zeros, and where $\mathbf{O}_{3 \times 1}$ is a column vector of three zeros; and

10 wherein means (d) generates vector \mathbf{g}_k for the k-th time moment in a form equivalent to:

$$\mathbf{g}_k = \mathbf{G}^T \mathbf{R}_k^{-1} \mathbf{Q}_k (\mathbf{Q}_k^T \mathbf{R}_k^{-1} \mathbf{Q}_k + q \mathbf{S}_k)^{-1} \mathbf{h}_k,$$

where:

$$\mathbf{h}_k = \left(1 - \frac{L_{RB}}{\|\mathbf{r}_k\|} \right) \mathbf{r}_k.$$

52. (Claims for Japan) A computer program to be installed in a computer for controlling the computer to perform the process of estimating a set of floating ambiguities associated with a set of phase measurements of a plurality n of satellite carrier signals made by a first navigation receiver (B) and a second navigation receiver (R) separated by a distance, wherein a baseline vector (x^o, y^o, z^o) relates the position of the second receiver to the first receiver, each satellite carrier signal being transmitted by a satellite and having a wavelength, wherein each receiver has a time clock for referencing its measurements and wherein any difference between the time clocks may be represented by an offset, said method receiving, for a plurality of two or more time moments j , the following inputs:

10 a vector γ_j^B representative of a plurality of pseudo-ranges measured by the first navigation receiver (B) and corresponding to the plurality of satellite carrier signals,

a vector γ_j^R representative of a plurality of pseudo-ranges measured by the second navigation receiver (R) and corresponding to the plurality of satellite carrier signals,

15 a vector D_j^B representative of a plurality of estimated distances between the satellites and the first navigation receiver (B),

a vector D_j^R representative of a plurality of estimated distances between the satellites and the second navigation receiver (R),

a vector φ_j^B representative of a plurality of full phase measurements of the satellite carrier signals measured by the first navigation receiver (B),

- 20 a vector $\boldsymbol{\varphi}_j^R$ representative of a plurality of full phase measurements of the satellite carrier signals measured by the second navigation receiver (R),
- a geometric Jacobian matrix \mathbf{H}_j^γ whose matrix elements are representative of the changes in the distances between the satellites and one of the receivers that would be caused by changes in that receiver's position and time clock offset, said process
- 25 comprising:
- (a) generating, for each time moment j , a vector $\Delta\boldsymbol{\gamma}_j$ of a plurality of range residuals of pseudo-range measurements made by the first and second navigation receivers in the form of:
$$\Delta\boldsymbol{\gamma}_j = (\boldsymbol{\gamma}_j^R - \boldsymbol{\gamma}_j^B) - (\mathbf{D}_j^R - \mathbf{D}_j^B);$$
- (b) generating, for each time moment j , a vector $\Delta\boldsymbol{\varphi}_j$ of a plurality of phase
- 30 residuals of full phase measurements made by the first and second navigation receivers in the form of:
- $$\Delta\boldsymbol{\varphi}_j = (\boldsymbol{\varphi}_j^R - \boldsymbol{\varphi}_j^B) - \boldsymbol{\Lambda}^{-1} \cdot (\mathbf{D}_j^R - \mathbf{D}_j^B),$$
- where $\boldsymbol{\Lambda}^{-1}$ is a diagonal matrix comprising the inverse wavelengths of the satellites;
- (c) generating, for time moment $j = 1$, an LU-factorization of a matrix \mathbf{M}_1 or a
- 35 matrix inverse of matrix \mathbf{M}_1 , the matrix \mathbf{M}_1 being a function of at least $\boldsymbol{\Lambda}^{-1}$ and \mathbf{H}_1^γ ;
- (d) generating, for time moment $j = 1$, a vector \mathbf{N}_1 as a function of at least $\Delta\boldsymbol{\gamma}_1$, $\Delta\boldsymbol{\varphi}_1$, and the LU-factorization of matrix \mathbf{M}_1 or the matrix inverse of matrix \mathbf{M}_1 ;
- (e) generating, for an additional time moment $j \neq 1$, an LU-factorization of a matrix \mathbf{M}_j or a matrix inverse of matrix \mathbf{M}_j , the matrix \mathbf{M}_j being a function of at least $\boldsymbol{\Lambda}^{-1}$,
- 40 \mathbf{H}_j^γ and an instance of matrix \mathbf{M} generated for a different time moment; and
- (f) generating, for an additional time moment $j \neq 1$, a vector \mathbf{N}_j as a function of at least $\Delta\boldsymbol{\gamma}_j$, $\Delta\boldsymbol{\varphi}_j$, and the LU-factorization or matrix \mathbf{M}_j or the matrix inverse of matrix \mathbf{M}_j , the vector \mathbf{N}_j having estimates of the floating ambiguities.

53. The computer program of Claim 52 wherein step (c) of the process comprises generating an LU-factorization for a matrix comprising a form equivalent to $(\mathbf{G}^T \mathbf{P}_1 \mathbf{G})$, where:

• the matrix \mathbf{G} has $2n$ rows, n columns, an upper sub-matrix, and a lower sub-matrix,
 5 one of the sub-matrices comprises an $n \times n$ zero matrix and the other sub-matrix comprising an $n \times n$ identity matrix,

• the matrix \mathbf{G}^T comprises the transpose matrix of matrix \mathbf{G} , and

• the matrix \mathbf{P}_1 has $2n$ rows, $2n$ columns, and a form which comprises a matrix

equivalent to $\left(\mathbf{R}_1^{-1} - \mathbf{R}_1^{-1} \mathbf{Q}_1 \left(\mathbf{Q}_1^T \mathbf{R}_1^{-1} \mathbf{Q}_1 + q \mathbf{S}_1 \right)^{-1} \mathbf{Q}_1^T \mathbf{R}_1^{-1} \right)$ where the matrix \mathbf{R}_1

10 is a weighting matrix, where the matrix \mathbf{R}_1^{-1} comprises an inverse of matrix \mathbf{R}_1 , where the matrix \mathbf{Q}_1 has an upper sub-matrix and a lower sub-matrix, one of the sub-matrices of \mathbf{Q}_1 comprising matrix \mathbf{H}_I^T and the other of the sub-matrices of \mathbf{Q}_1 comprising the matrix product $\mathbf{\Lambda}^{-1} \mathbf{H}_I^T$, and wherein the matrix \mathbf{Q}_1^T comprises the transpose of matrix \mathbf{Q}_1 , and where the quantity $q \mathbf{S}_k$ is a zero matrix when the distance
 15 between the first and second navigation receivers is unconstrained and where q may be a non-zero weighting parameter and \mathbf{S}_k may be a non-zero matrix when the distance between the first and second navigation receivers is constrained.

54. The computer program of Claim 53 wherein step (d) of the process comprises the step of generating vector \mathbf{N}_1 to comprise a vector having a form equivalent to

$\mathbf{M}_1^{-1} \left(\mathbf{G}^T \mathbf{P}_1 \boldsymbol{\mu}_1 + q \mathbf{g}_1 \right)$, where the matrix \mathbf{M}_1^{-1} comprises an inverse of matrix of matrix \mathbf{M}_1 , and where the vector $\boldsymbol{\mu}_1$ comprises the vector $[\Delta \gamma_1, \Delta \varphi_1]^T$, and where the quantity
 5 $q \mathbf{g}_k$ is a zero vector when the distance between the first and second navigation receivers is unconstrained and where q may be non-zero and \mathbf{g}_k may be a non-zero vector when the distance between the first and second navigation receivers is constrained.

55. The computer program of Claim 52 wherein step (e) of the process comprises generating an LU-factorization for a matrix comprising a form equivalent to

$\mathbf{M}_j = \mathbf{M}_{j-1} + \mathbf{G}^T \mathbf{P}_j \mathbf{G}$, where:

• \mathbf{M}_{j-1} comprises the matrix \mathbf{M}_1 of step (c) when $j = 2$ and comprises the matrix \mathbf{M}_j of
 5 step (e) for the $j-1$ time moment when $j > 2$,

- the matrix \mathbf{G} has $2n$ rows, n columns, an upper sub-matrix, and a lower sub-matrix, one of the sub-matrices comprising an $n \times n$ zero matrix and the other sub-matrix comprising an $n \times n$ identity matrix,

- the matrix \mathbf{G}^T comprises the transpose matrix of matrix \mathbf{G} , and

- 10 • the matrix \mathbf{P}_j has $2n$ rows, $2n$ columns, and a form which comprise a matrix

equivalent to $\left(\mathbf{R}_j^{-1} - \mathbf{R}_j^{-1} \mathbf{Q}_j (\mathbf{Q}_j^T \mathbf{R}_j^{-1} \mathbf{Q}_j + q\mathbf{S}_j)^{-1} \mathbf{Q}_j^T \mathbf{R}_j^{-1} \right)$ where the matrix

\mathbf{R}_j is a weighting matrix, where the matrix \mathbf{R}_j^{-1} comprises an inverse of matrix \mathbf{R}_j , where the matrix \mathbf{Q}_j has an upper sub-matrix and a lower sub-matrix, one of the sub-matrices of \mathbf{Q}_j comprising matrix \mathbf{H}_j^T and the other of the sub-matrices of \mathbf{Q}_j

- 15 comprising the matrix product $\mathbf{\Lambda}^{-1} \mathbf{H}_j^T$, and wherein the matrix \mathbf{Q}_j^T comprises the transpose of matrix \mathbf{Q}_j , and where the quantity $q\mathbf{S}_j$ is a zero matrix when the distance between the first and second navigation receivers is unconstrained and where q may be a non-zero weighting parameter and \mathbf{S}_j may be a non-zero matrix when the distance between the first and second navigation receivers is constrained.

20

56. The computer program of Claim 55 wherein step (f) of the process comprises the step of generating vector \mathbf{N}_j to comprise a vector having a form equivalent to

$\mathbf{N}_{j-1} + \mathbf{M}_j^{-1} \left[\mathbf{G}^T \mathbf{P}_j (\boldsymbol{\mu}_j - \mathbf{G} \mathbf{N}_{j-1}) + q\mathbf{g}_j \right]$, where the matrix \mathbf{M}_j^{-1} comprises an

inverse of matrix of matrix \mathbf{M}_j , where the vector $\boldsymbol{\mu}_j$ comprises the vector $[\Delta\gamma_j, \Delta\varphi_j]^T$, and

- 5 where the vector \mathbf{N}_{j-1} comprises the vector \mathbf{N}_1 generated by step (d) when $j = 2$ and comprises the vector \mathbf{N}_{j-1} generated by step (f) for the $j-1$ time moment when $j > 2$, and where the quantity $q\mathbf{g}_j$ is a zero vector when the distance between the first and second navigation receivers is unconstrained and where q may be non-zero and \mathbf{g}_j may be a non-zero vector when the distance between the first and second navigation receivers is
- 10 constrained.

57. The computer program of claim 56 wherein the distance between the first and second navigation receivers is constrained to a distance L_{RB} , wherein step (c) of the process generates matrix \mathbf{S}_1 in a form equivalent to:

$$\mathbf{S}_1 = \left(1 - \frac{L_{RB}}{\|\mathbf{r}_1\|}\right) \begin{pmatrix} \mathbf{I}_3 & \mathbf{O}_{3 \times 1} \\ \mathbf{O}_{1 \times 3} & 0 \end{pmatrix} + \frac{L_{RB}}{\|\mathbf{r}_1\|} \mathbf{r}_1 \mathbf{r}_1^T$$

5 where \mathbf{r}_1 is a vector comprising estimates of the three coordinates of the baseline vector for the time moment $j=1$ and a zero as fourth component, where \mathbf{r}_1^T is the vector transpose of \mathbf{r}_1 , where \mathbf{I}_3 is the 3-by-3 identity matrix, where $\mathbf{O}_{1 \times 3}$ is a row vector of three zeros, and where $\mathbf{O}_{3 \times 1}$ is a column vector of three zeros; and

wherein step (d) generates vector \mathbf{g}_1 for the time moment $j=1$ in a form equivalent

10 to:

$$\mathbf{g}_1 = \mathbf{G}^T \mathbf{R}_1^{-1} \mathbf{Q}_1 (\mathbf{Q}_1^T \mathbf{R}_1^{-1} \mathbf{Q}_1 + q \mathbf{S}_1)^{-1} \mathbf{h}_1,$$

where:

$$\mathbf{h}_1 = \left(1 - \frac{L_{RB}}{\|\mathbf{r}_1\|}\right) \mathbf{r}_1.$$

58. The computer program of claim 56 wherein the distance between the first and second navigation receivers is constrained to a distance L_{RB} , wherein step (e) of the process generates matrix \mathbf{S}_j in a form equivalent to:

$$\mathbf{S}_j = \left(1 - \frac{L_{RB}}{\|\mathbf{r}_j\|}\right) \begin{pmatrix} \mathbf{I}_3 & \mathbf{O}_{3 \times 1} \\ \mathbf{O}_{1 \times 3} & 0 \end{pmatrix} + \frac{L_{RB}}{\|\mathbf{r}_j\|} \mathbf{r}_j \mathbf{r}_j^T$$

5 where \mathbf{r}_j is a vector comprising estimates of the three coordinates of the baseline vector for the j -th time moment and a zero as fourth vector component, where \mathbf{r}_j^T is the vector transpose of \mathbf{r}_j , where \mathbf{I}_3 is the 3-by-3 identity matrix, where $\mathbf{O}_{1 \times 3}$ is a row vector of three zeros, and where $\mathbf{O}_{3 \times 1}$ is a column vector of three zeros; and

wherein step (f) generates vector \mathbf{g}_j for the j -th time moment in a form equivalent

10 to:

$$\mathbf{g}_j = \mathbf{G}^T \mathbf{R}_j^{-1} \mathbf{Q}_j (\mathbf{Q}_j^T \mathbf{R}_j^{-1} \mathbf{Q}_j + q \mathbf{S}_j)^{-1} \mathbf{h}_j,$$

where:

$$\mathbf{h}_j = \left(1 - \frac{L_{RB}}{\|\mathbf{r}_j\|}\right) \mathbf{r}_j.$$

59. The computer program of Claim 55 wherein the generation of the LU-factorization in step (e) of the process comprises the steps of:

(g) generating an LU-factorization of matrix \mathbf{M}_{j-1} in a form equivalent to $\mathbf{L}_{j-1} \mathbf{L}_{j-1}^T$ wherein \mathbf{L}_{j-1} is a low-triangular matrix and \mathbf{L}_{j-1}^T is the transpose of \mathbf{L}_{j-1} ;

5 (h) generating a factorization of $\mathbf{G}^T \mathbf{P}_j \mathbf{G}$ in a form equivalent to $\mathbf{T}_j \mathbf{T}_j^T = \mathbf{G}^T \mathbf{P}_j \mathbf{G}$, where \mathbf{T}_j^T is the transpose of \mathbf{T}_j ;

(i) generating an LU-factorization of matrix \mathbf{M}_j in a form equivalent to $\mathbf{L}_j \mathbf{L}_j^T$ from a plurality n of rank-one modifications of matrix \mathbf{L}_{j-1} , each rank-one modification being based on a respective column of matrix \mathbf{T}_j , where n is the number of rows in matrix \mathbf{M}_j .

60. The computer program of Claim 59 wherein step (h) of the process generates matrix \mathbf{T}_j from a Cholesky factorization of $\mathbf{G}^T \mathbf{P}_j \mathbf{G}$.

61. The computer program of Claim 55 wherein step (d) of the process comprises generating a vector \mathbf{B}_1 to comprise a vector having a form equivalent to $\mathbf{G}^T \mathbf{P}_1 \boldsymbol{\mu}_1 + q\mathbf{g}_1$, where the vector $\boldsymbol{\mu}_1$ comprises the vector $[\Delta\gamma_1, \Delta\phi_1]^T$, and where the quantity $q\mathbf{g}_j$ is a zero vector when the distance between the first and second navigation receivers is
5 unconstrained and where q may be non-zero and \mathbf{g}_j may be a non-zero vector when the distance between the first and second navigation receivers is constrained; and

wherein step (f) of the process further comprises generating, for each time moment $j \neq 1$, a vector \mathbf{B}_j to comprise a matrix having a form equivalent to $\mathbf{B}_{j-1} + \mathbf{G}^T \mathbf{P}_j \boldsymbol{\mu}_j + q\mathbf{g}_j$, where the vector $\boldsymbol{\mu}_j$ comprises the vector $[\Delta\gamma_j, \Delta\phi_j]^T$, and where the vector \mathbf{B}_{j-1} is the
10 vector \mathbf{B}_1 generated by step (d) when $j = 2$ and comprises the vector generated by step (f) for the for the $j-1$ time moment when $j > 2$, and where the quantity $q\mathbf{g}_j$ is a zero vector when the distance between the first and second navigation receivers is unconstrained and where q may be non-zero and \mathbf{g}_j may be a non-zero vector when the distance between the first and second navigation receivers is constrained; and

15 wherein step (f) of the process further comprises generating vector \mathbf{N}_j to comprise a vector having a form equivalent to $\mathbf{N}_j = [\mathbf{M}_j]^{-1} \mathbf{B}_j$, where the matrix \mathbf{M}_j^{-1} comprises an inverse of matrix of matrix \mathbf{M}_j .

62. The computer program of claim 61 wherein the distance between the first and second navigation receivers is constrained to a distance L_{RB} , wherein step (c) of the process generates matrix S_1 in a form equivalent to:

$$S_1 = \left(1 - \frac{L_{RB}}{\|\mathbf{r}_1\|} \right) \begin{pmatrix} \mathbf{I}_3 & \mathbf{O}_{3 \times 1} \\ \mathbf{O}_{1 \times 3} & 0 \end{pmatrix} + \frac{L_{RB}}{\|\mathbf{r}_1\|} \mathbf{r}_1 \mathbf{r}_1^T$$

5 where \mathbf{r}_1 is a vector comprising estimates of the three coordinates of the baseline vector for the time moment $j=1$ and a zero as fourth component, where \mathbf{r}_1^T is the vector transpose of \mathbf{r}_1 , where \mathbf{I}_3 is the 3-by-3 identity matrix, where $\mathbf{O}_{1 \times 3}$ is a row vector of three zeros, and where $\mathbf{O}_{3 \times 1}$ is a column vector of three zeros; and

10 wherein step (d) generates vector \mathbf{g}_1 for the time moment $j=1$ in a form equivalent to:

$$\mathbf{g}_1 = \mathbf{G}^T \mathbf{R}_1^{-1} \mathbf{Q}_1 (\mathbf{Q}_1^T \mathbf{R}_1^{-1} \mathbf{Q}_1 + q S_1)^{-1} \mathbf{h}_1,$$

where:

$$\mathbf{h}_1 = \left(1 - \frac{L_{RB}}{\|\mathbf{r}_1\|} \right) \mathbf{r}_1.$$

63. The computer program of claim 61 wherein the distance between the first and second navigation receivers is constrained to a distance L_{RB} , wherein step (e) of the process generates matrix S_j in a form equivalent to:

$$S_j = \left(1 - \frac{L_{RB}}{\|\mathbf{r}_j\|} \right) \begin{pmatrix} \mathbf{I}_3 & \mathbf{O}_{3 \times 1} \\ \mathbf{O}_{1 \times 3} & 0 \end{pmatrix} + \frac{L_{RB}}{\|\mathbf{r}_j\|} \mathbf{r}_j \mathbf{r}_j^T$$

5 where \mathbf{r}_j is a vector comprising estimates of the three coordinates of the baseline vector for the j -th time moment and a zero as fourth vector component, where \mathbf{r}_j^T is the vector transpose of \mathbf{r}_j , where \mathbf{I}_3 is the 3-by-3 identity matrix, where $\mathbf{O}_{1 \times 3}$ is a row vector of three zeros, and where $\mathbf{O}_{3 \times 1}$ is a column vector of three zeros; and

10 wherein step (f) of the process generates vector \mathbf{g}_j for the j -th time moment in a form equivalent to:

$$\mathbf{g}_j = \mathbf{G}^T \mathbf{R}_j^{-1} \mathbf{Q}_j (\mathbf{Q}_j^T \mathbf{R}_j^{-1} \mathbf{Q}_j + q S_j)^{-1} \mathbf{h}_j,$$

where:

$$\mathbf{h}_j = \left(1 - \frac{L_{RB}}{\|\mathbf{r}_j\|} \right) \mathbf{r}_j.$$

64. (Claims for Japan) A computer program to be installed in a computer for controlling the computer to perform the process of estimating a set of floating ambiguities associated with a set of phase measurements of a plurality n of satellite carrier signals made by a first navigation receiver (B) and a second navigation receiver (R), wherein a
- 5 baseline vector (x^o, y^o, z^o) relates the position of the second receiver to the first receiver, each satellite carrier signal being transmitted by a satellite and having a wavelength, wherein each receiver has a time clock for referencing its measurements and wherein any difference between the time clocks may be represented by an offset, said method receiving, for a plurality of two or more time moments j , the following inputs for each
- 10 time moment j :
- a vector γ_j^B representative of a plurality of pseudo-ranges measured by the first navigation receiver (B) and corresponding to the plurality of satellite carrier signals,
 - a vector γ_j^R representative of a plurality of pseudo-ranges measured by the second navigation receiver (R) and corresponding to the plurality of satellite carrier signals,
 - 15 a vector D_j^B representative of a plurality of estimated distances between the satellites and the first navigation receiver (B),
 - a vector D_j^R representative of a plurality of estimated distances between the satellites and the second navigation receiver (R),
 - a vector ϕ_j^B representative of a plurality of full phase measurements of the
 - 20 satellite carrier signals measured by the first navigation receiver (B),
 - a vector ϕ_j^R representative of a plurality of full phase measurements of the satellite carrier signals measured by the second navigation receiver (R),
 - a geometric Jacobian matrix H_j^γ whose matrix elements are representative of the changes in the distances between the satellites and one of the receivers that would be
 - 25 caused by changes in that receiver's position and time clock offset, the process comprising:

(a) generating, for each time moment j , a vector $\Delta \gamma_j$ of a plurality of range residuals of pseudo-range measurements made by the first and second navigation receivers in the form of: $\Delta \gamma_j = (\gamma_j^R - \gamma_j^B) - (D_j^R - D_j^B)$, said step generating a set of
 30 range residuals $\Delta \gamma_k$, $k=1, \dots, j$;

(b) generating, for each time moment j , a vector $\Delta \varphi_j$ of a plurality of phase residuals of full phase measurements made by the first and second navigation receivers in the form of: $\Delta \varphi_j = (\varphi_j^R - \varphi_j^B) - \Lambda^{-1} \cdot (D_j^R - D_j^B)$, where Λ^{-1} is a diagonal matrix comprising the inverse wavelengths of the satellites, said step generating a set of phase
 35 residuals $\Delta \varphi_k$, $k=1, \dots, j$;

(c) generating an LU-factorization of a matrix \mathbf{M} or a matrix inverse of matrix \mathbf{M} , the matrix \mathbf{M} being a function of at least Λ^{-1} and \mathbf{H}_k^γ , for index k of \mathbf{H}_k^γ covering at least two of the time moments j ;

(d) generating a vector \mathbf{N} of estimated floating ambiguities as a function of at least
 40 the set of range residuals $\Delta \gamma_k$, the set of phase residuals $\Delta \varphi_k$, and the LU-factorization of matrix \mathbf{M} or the matrix inverse of matrix \mathbf{M} .

65. The computer program of Claim 64 wherein step (c) of the process comprises generating matrix \mathbf{M} in a form equivalent to the summation $\left[\sum_{k=1}^j (\mathbf{G}^T \mathbf{P}_k \mathbf{G}) \right]$,

where:

- the matrix \mathbf{G} has $2n$ rows, n columns, an upper sub-matrix, and a lower sub-matrix,
 5 one of the sub-matrices comprises an $n \times n$ zero matrix and the other sub-matrix comprising an $n \times n$ identity matrix,
- the matrix \mathbf{G}^T comprises the transpose matrix of matrix \mathbf{G} , and
- the matrix \mathbf{P}_k has $2n$ rows, $2n$ columns, and a form which comprises a matrix
 equivalent to $\mathbf{P}_k = \mathbf{R}_k^{-1} - \mathbf{R}_k^{-1} \mathbf{Q}_k (\mathbf{Q}_k^T \mathbf{R}_k^{-1} \mathbf{Q}_k + q \mathbf{S}_k)^{-1} \mathbf{Q}_k^T \mathbf{R}_k^{-1}$, where the
 10 matrix \mathbf{R}_k is a weighting matrix, where the matrix \mathbf{R}_k^{-1} comprises an inverse of matrix \mathbf{R}_k , where the matrix \mathbf{Q}_k has an upper sub-matrix and a lower sub-matrix, one of the sub-matrices of \mathbf{Q}_k comprising matrix \mathbf{H}_k^γ and the other of the sub-matrices of \mathbf{Q}_k

15 comprising the matrix product $\Lambda^{-1} H_k^T$, and wherein the matrix Q_k^T comprises the transpose of matrix Q_k , and where the quantity qS_k is a zero matrix when the distance between the first and second navigation receivers is unconstrained and where q may be a non-zero weighting parameter and S_k may be a non-zero matrix when the distance between the first and second navigation receivers is constrained.

66. The computer program of Claim 65 wherein step (d) of the process comprises generating matrix N in a form equivalent to:

$$N = M^{-1} \times \left[\sum_{k=1}^j (G^T P_k \mu_k + q g_k) \right],$$

5 where the matrix M^{-1} comprises an inverse of matrix of matrix M , where the vector μ_k comprises the vector $[\Delta \gamma_k, \Delta \phi_k]^T$, and where the quantity $q g_k$ is a zero vector when the distance between the first and second navigation receivers is unconstrained and where q may be non-zero and g_k may be a non-zero vector when the distance between the first and second navigation receivers is constrained.

67. The computer program of claim 66 wherein the distance between the first and second navigation receivers is constrained to a distance L_{RB} , wherein step (c) of the process generates matrix S_k for the k -th time moment in a form equivalent to:

$$S_k = \left(1 - \frac{L_{RB}}{\|r_k\|} \right) \begin{pmatrix} I_3 & O_{3 \times 1} \\ O_{1 \times 3} & 0 \end{pmatrix} + \frac{L_{RB}}{\|r_k\|} r_k r_k^T$$

5 where r_k is a vector comprising estimates of the three coordinates of the baseline vector at the k -th time moment, and a zero as fourth component, where r_k^T is the vector transpose of r_k , where I_3 is the 3-by-3 identity matrix, where $O_{1 \times 3}$ is a row vector of three zeros, and where $O_{3 \times 1}$ is a column vector of three zeros; and

10 wherein step (d) of the process generates vector g_k for the k -th time moment in a form equivalent to:

$$g_k = G^T R_k^{-1} Q_k (Q_k^T R_k^{-1} Q_k + q S_k)^{-1} h_k,$$

where:

$$\mathbf{h}_k = \left(1 - \frac{L_{RB}}{\|\mathbf{r}_k\|} \right) \mathbf{r}_k.$$

68. The computer program of Claim 65 wherein at least one of the weighting matrices \mathbf{R}_k comprises an identity matrix multiplied by a scalar quantity.